1. Let $V$ be a real vector space. Prove that:
   
   (a) Prove that for $x \in V$, $0 \cdot x = \bar{0}$.
   
   (b) Prove that $\lambda x = \bar{0}$ if and only if $x = \bar{0}$ or $\lambda = 0$.
   
   (c) Prove that $(-1)x = -x$.

2. Problem 2.2 from Jänisch

3. Problem 2.3 from Jänisch

4. (a) Prove that all the functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the differential equation
   \[ f''(x) - f(x) = 0 \]
   form a vector space.

   (b) Prove that the solutions to this differential equation satisfying $f(\pi) = 0$ form its linear subspace.

   (c) Now consider the differential equation $f'(x) = f(x)^2$. Do its solutions form a linear subspace of the space of all differentiable functions?

5. (a) Let $S$ be the set of all functions $f : [0, 1] \to \mathbb{R}$ such that $f$ is integrable and $\int_0^1 f(x) \, dx = 0$. Is the set $S$ a linear subspace of the space of all functions?

   (b) Let $W$ be the set of integrable functions from $[0, 1] \to \mathbb{R}$ defined by the condition $\int_0^1 f(x) \, dx = 1$. Is the set $W$ a linear subspace of the space of all functions?

6. Find all the complex numbers $z$ such that $z^3 = 1$.

7. Prove that the set $F := \{a + b\sqrt{5} | a, b \in \mathbb{Q}\} \subset \mathbb{R}$ forms a field (with respect to the operations of addition and multiplication that it inherits from $\mathbb{R}$) (it is denoted by $\mathbb{Q}(\sqrt{5})$ ).