Homework 5: Linear transformations; matrices. Part 2.

1. Consider the linear space V polynomials of degree not greater than n over a field $F \subset \mathbb{C}$ (that is, the space of all functions $f : F \to F$ of the form $f(x) = a_n x^n + \cdots + a_1 x + a_0$, where $a_0, \ldots, a_n \in F$). Let $D : V \to V$ be the linear map $D(f) = f'$ (the derivative). Find the kernel and image of D.

2. Let V be an arbitrary vector space over a field F, and let $P : V \to V$ be a linear operator with the property that $P^2 = P$ (here by P^2 we mean P composed with itself). Such linear operators are called *projectors*.

   (a) Prove that $V = \mathrm{Ker}(P) \oplus \mathrm{Im}(P)$.

   (b) Make an example of such a linear operator on $\mathbb{R}^3$.

3. Problem 5.1 from Jänisch

4. Problem 5.2 from Jänisch

5. Problem 7.1 from Jänisch. In addition, find a basis for the space $\mathrm{Ker}(A)$ for the matrix of this system of equations.

6. Problem 7.2 from Jänisch. In addition, find a basis for the space $\mathrm{Ker}(A)$ for the matrix of this system of equations.