Homework 6: Determinants.
You do not have to type the matrices – handwritten and scanned will be accepted.

1. Problem 5.3 from Jänisch

2. Compute the following determinants:

   (a)
$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

   (b)
$$\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{vmatrix}.$$

3. Problem 6.2 from Jänisch

4. Find the $n \times n$ -determinant
$$\begin{vmatrix} \lambda & 1 & 0 & \dots & 0 \\ 0 & \lambda & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & \lambda & 1 \\ 0 & \dots & 0 & 0 & \lambda \end{vmatrix}$$
   where λ is a scalar (it should be a function of λ).

5. Let $a_0, \dots, a_{n-1}$ be scalars. Prove that
$$\begin{vmatrix} -t & 0 & 0 & \dots & 0 & -a_0 \\ 1 & -t & 0 & \dots & 0 & -a_1 \\ 0 & 1 & -t & \dots & 0 & -a_2 \\ \dots & \dots & \dots & \dots & \dots & \\ 0 & \dots & 0 & 1 & -t & -a_{n-2} \\ 0 & \dots & & \dots & 1 & -(t + a_{n-1}) \end{vmatrix}$$
   equals $(-1)^n(t^n + a_{n-1}t^{n-1} + \dots + a_1 t + a_0)$ (as a function of t).

   Hint: Use the cofactor expansion with respect to the last column.