Homework 7: Euclidean spaces

1. Problem 8.1 from Janisch

2. Problem 8.2 from Jänisch, but there is a typo in the matrix $A$ – it should be symmetric, i.e., $a_{23}$ should be 1, not 0)

3. (a) Prove that for any real numbers $a_1, \ldots, a_n$ satisfying $a_1 + \cdots + a_n = 1$, we have

$$a_1^2 + \cdots + a_n^2 \geq \frac{1}{n}.$$ 

(b) Find the minimum of the function $f(x_1, \ldots, x_n) = x_1^2 + \cdots + x_n^2$ on the hyperplane $x_1 + \cdots + x_n = 1$.

4. Find the maximum of the function $x + 2y + 3z$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

5. Problem 2 chapter 15 (p. 129) from Curtis

6. Problem 6 chapter 15 (p.129) from Curtis