Homework 9: Eigenvalues.

1. Problem 9.1 from Jänisch

2. Problem 9.2 from Jänisch

3. Problem 9.3 from Jänisch

4. Problem 9.1* from Jänisch

5. Let $a_0, \ldots, a_{n-1}$ be scalars. Find the characteristic polynomial of the matrix
$$\begin{vmatrix} 0 & 0 & 0 & \ldots & 0 & -a_0 \\ 1 & 0 & 0 & \ldots & 0 & -a_1 \\ 0 & 1 & 0 & \ldots & 0 & -a_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \\ 0 & \ldots & 0 & 1 & 0 & -a_{n-2} \\ 0 & \ldots & & \ldots & 1 & -a_{n-1}) \end{vmatrix}.$$

   Hint. Look at the last problem of Homework 6.

   Remark. This shows that any polynomial arises as the characteristic polynomial of some matrix (in case you were wondering). This matrix is called the *Frobenius companion matrix* for a given polynomial.

6. (a) Problem 10.1 from Janisch

   (b) Find A^5 for the matrix A from this problem.

7. (a) Let $A : V \to V$ be a self-adjoint linear operator on a vector space V. Prove that if U is an invariant subspace for A, then $U^{\perp}$ is also invariant under A.

   (b) Problem 10.3 from Jänisch

8. Let J_λ be the 3×3 Jordan block with the eigenvalue λ:
$$J_\lambda = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}.$$

   Find J_λ^2, J_λ^3 and J_λ^4.

   Hint: It might help to use the binomial formula for matrices.