Cartesian product of sets

Def: A, B - sets.

$$A \times B = \{ (a,b) \mid a \in A,\ b \in B \}.$$

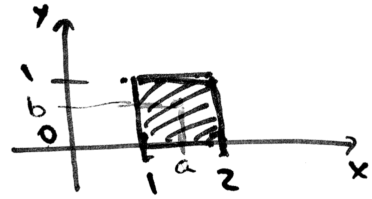
times

ordered pairs of elements.

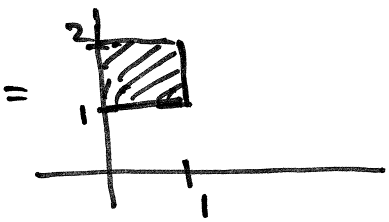
Example $A = [1,2]$, $B = [0,1]$

interval in $\mathbb{R}$

$A \times B =$



$B \times A =$



pictures of $A \times B$ as a subset of $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

$= \{ (x,y) \mid x \in \mathbb{R},\ y \in \mathbb{R} \}.$

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ - familiar "3-space"

Def: $A_1, \dots, A_n$ - a finite collection of sets.

$$A_1 \times \dots \times A_n \overset{\text{def}}{=} \{ \underbrace{(a_1, \dots, a_n)}_{n\text{-tuple}} \mid a_i \in A_i \}.$$

Functions (Maps)

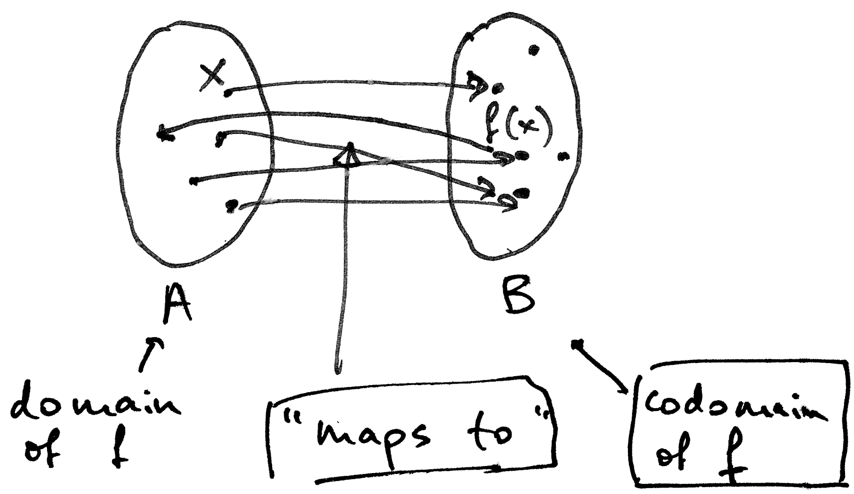
$f: A \to B$ - a function is an assignment of <u>one</u> element of B (called $f(x)$) to every element x of A.

domain of the function f.

codomain

Today: functions

$f : A \to B$



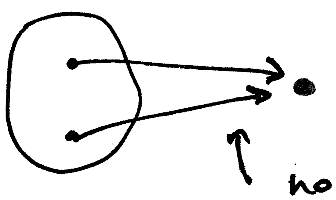
Function
for each element of A
there has to be
exactly one
outgoing arrow

$x \longmapsto f(x)$

$f(x)$ is called the image of x.

Def: ① $f : A \to B$ is called injective if

$$x \neq y \implies f(x) \neq f(y)$$

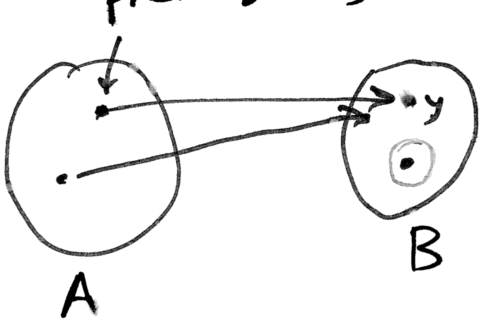


not injective

"distinct elements of A have distinct images"

② A function $f : A \to B$ is called surjective if every element of B has a preimage (inverse image) in A:

preimage of y



← no "lonely" dots in B.