**Cartesian product of sets**

**Def:** $A, B$ - sets.

\[
A \times B = \{ (a, b) \mid a \in A, b \in B \}.
\]

\[\times\]  \text{ordered pairs of elements.}

**Example**

\[A = [1, 2], \quad B = [0, 1]\]

\[\text{Interval in IR}\]

\[A \times B = \quad B \times A =\]

\[\begin{array}{cc}
\text{pictures of } A \times B \text{ as a subset of } IR^2 & = \{ (x, y) \mid x \in IR, y \in IR \} \\
\end{array}\]

\[\text{IR}^3 = \text{IR} \times \text{IR} \times \text{IR} - \text{familiar "3-space"}\]

**Def:** $A_1, \ldots, A_n$ - a finite collection of sets.

\[A_1 \times \ldots \times A_n \equiv \{ (a_1, \ldots, a_n) \mid a_i \in A_i \}, \quad \text{\textit{n-tuple}}\]

---

**Functions (Maps)**

\[f : A \rightarrow B \text{ - a function}\]

\[\uparrow \quad \uparrow\]

\text{domain} \quad \text{codomain}

\text{of the function } f.

\text{is an assignment of one element of } B \text{ (called } f(x) \text{) to every element } x \text{ of } A.
Today: Functions

\[ f : A \rightarrow B \]

Function for each element of \( A \) there has to be exactly one outgoing arrow \( x \mapsto f(x) \)

\( f(x) \) is called the image of \( x \).

**Def:** 1. \( f : A \rightarrow B \) is called injective if
\[
    x \neq y \implies f(x) \neq f(y)
\]

"distinct elements of \( A \) have distinct images" 

\( f \) is not injective.

2. A function \( f : A \rightarrow B \) is called surjective if every element of \( B \) has a preimage (inverse image) preimage of \( y \) in \( A \):

\( f \) has no "lonely" dots in \( B \).