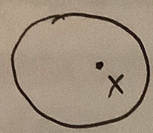
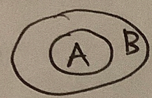
Venn diagrams



← set A, x - an element of A



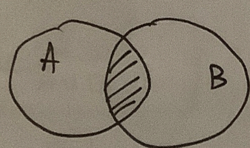
- "A is a subset of B".

Def: $A \cap B$ - the intersection of A and B

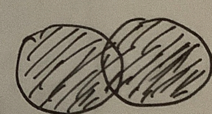
$= \{x \mid x \in A \text{ and } x \in B\}$.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

↑ includes when $x \in A \cap B$.

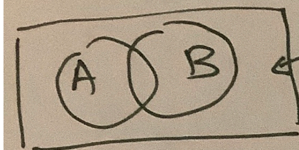


$A \cap B$



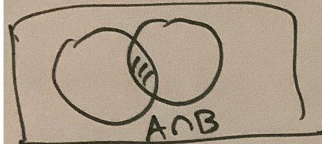
$A \cup B$

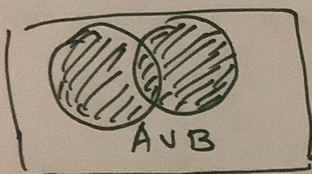
More on unions and intersections

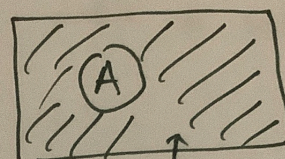


← universe (universal set). ← meant in order to avoid "set of all sets"

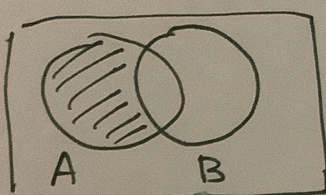
(Exer: think about if you had the set of all sets, would it be an element of itself?







↑ the complement of A (denoted by $\overline{A}$) (needs the universal set to be precise)



Def: difference of sets: $A \setminus B$ ← \setminus

$$A \setminus B = \{ x : x \in A,\ x \notin B \}.$$

$$= A \cap \overline{B}.$$

Statements and sets

Let U be a universal set.

For any subset $A \subset U$, we can make a statement about an element $x \in U$:

$P_A(x)$ = " $x \in A$ " ← true if x is an element of A

false if x is not in A.

A statement like this one whose truth depends on a variable x is called a **predicate**

We just associated with a set A a **predicate** $P_A(x)$.

Conversely, given a predicate $Q(x)$, one can define the set $A_Q(x)$:

$$A_Q(x) = \{ x \in U \mid Q(x) \text{ is True} \}$$

(↑ U: the universal set)

Then we have a correspondence between the operations: (check this!)

$A \cup B$: $P_A(x) \vee P_B(x)$ (↑ $\vee$: or)

$A \cap B$: $P_A(x) \wedge P_B(x)$

$\overline{A}$: $\neg P_A(x)$ - negation

One can define operations on statements using the Truth Tables:

for example, $P \vee Q$ is defined as:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

← these truth values define $P \vee Q$ (a new statement "P or Q")

↑ ↑ all possibilities for T/F for P and Q

- This way, one can define the implication $P \Rightarrow Q$: (it is a new statement, "P implies Q")

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- How to negate an implication

$\neg(P \Rightarrow Q)$ is: $P \wedge \neg Q$

("P holds but Q doesn't")

(it is NOT an implication!)