Review topics for the final exam

- Everything that was on the list of topics for the midterm.
- Definition of the group action on a set; examples (most importantly, the action of the group on itself by conjugations); orbits and stabilizers. Formulas relating the sizes of orbits and stabilizers.
- Conjugacy classes in the group S_n.
- Rings: basic definitions and examples: the rings $\mathbb{Z}/n\mathbb{Z}$, polynomial rings. Homomorphisms of rings.
- The notion of units and zero divisors. Definition of a domain. Examples.
- The notion of an ideal. Examples. The notion of a set of generators for an ideal. Examples of principal and non-principal ideals.
- Quotient rings and Isomorphism theorem.
- $\mathbb{Z}$ is a PID, $F[x]$ is a PID if F is a field (including Euclidean algorithm for polynomials).
- $\mathbb{Z}[i]$ is a PID (with proof).
- Examples of rings that are not principal ideal domains.
- Prime and maximal ideals.
- An ideal I in R is maximal iff R/I is a field; I is prime iff R/I is a domain.
- Irreducible polynomials and maximal ideals in a ring $F[x]$, where F is a field.
- Finite fields: existence, some basic calculations involving roots of polynomials in a finite field.
- The notion of prime and irreducible elements in an arbitrary domain.
- Examples of rings where unique factorization fails.
- Euclidean domains, unique factorization.