Problem Set 2. Due Thursday January 23.

1. Section 7.2, Problem 3. (p. 238)

2. Section 7.3, Problem 17.

3. Section 7.3, Problem 2.

4. Section 7.3, Problem 6.

5. Section 7.3, Problem 10.

6. Section 7.3, Problem 14 (hint: it might help to do Problem 13 first).

7. Section 7.3, Problem 19.

8.* Section 7.3, Problem 21.

9. Section 7.3, Problem 24.

10. Let $R = \mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$.
Let $I = \{a + b\sqrt{5} \in R \mid a - b$ is divisible by $4\}$. Decide whether I is an ideal in R, and if yes, find the quotient ring R/I.

11. Let $\mathbb{Q}[\pi]$ be the set of all real numbers of the form

$$r_0 + r_1\pi + \cdots + r_n\pi^n \text{ with } r_i \in \mathbb{Q}, n \geq 0.$$

It is easy to show that $\mathbb{Q}[\pi]$ is a subring or $\mathbb{R}$ (you don't have to write the proof of this). Is $\mathbb{Q}[\pi]$ isomorphic to the polynomial ring $\mathbb{Q}[x]$?