List of topics for the Midterm

(1) The definitions of a ring, commutative ring, ring with identity, homomorphism and isomorphism of rings.
(2) Key examples: the quaternions, quadratic integer rings, polynomial rings, rings of functions on a set, matrix rings.
(3) Properties of elements in rings: units, zero divisors, nilpotent elements. Examples of these types of elements in the rings listed above. Integral domains.
(4) Ideals – the definition; the notion of a quotient ring.
(5) The notion of generators of an ideal; need to know the definitions of a sum/product/intersection of ideals, inclusions between these, and be able to compute some examples. Principal ideals; examples of non-principal ideals.
(6) The four isomorphism theorems. Be prepared for questions such as "find such-and-such quotient ring", which means, identify the quotient R/I for some ring R and an ideal $I \subset R$ as one of the familiar examples. Also, be ready for questions such as "are these two rings isomorphic?" (similar to homework, e.g. $\mathbb{Z}[i]$ is not isomorphic to $\mathbb{Z}[\sqrt{2}]$).
(7) The notions of prime and maximal ideal. The criterion for an ideal to be prime/maximal. Why a maximal ideal is always prime. Examples of prime ideals that are not maximal.
(8) Prime and irreducible elements. Factorization into irreducibles. Examples in polynomial rings and quadratic integer rings. Be prepared for questions such as, "is such-and-such element of $\mathbb{Z}[\sqrt{-11}]$ irreducible" (or prime)?
(9) Euclidean domains, principal ideal domains, unique factorization domains; inclusions between these classes of rings. Also, need to know examples of rings that belong to one class but not the other (for example, PID but not Euclidean, etc.)
(10) Need to know some basic examples in quadratic integer rings, in particular, be able to detect (for small values of D that we have discussed) if the given quadratic integer ring has unique factorization or not.
(11) The ring of polynomials over a field is Euclidean (division algorithm for polynomials).
(12) Chinese Remainder Theorem for rings, and consequences, especially for polynomial rings.
(13) The theorem that R is a UFD if and only if $R[x]$ is a UFD (the last topic almost covered before the break).