Problem Set 3. Due Tuesday February 28.

All references are to Atiyah-Macdonald.

(1) Exercise 3 on p.31.
(2) Exercise 4 on p.31.
(3) Exercise 8 on p.32.
(4) Exercise 13 on p.32.
(5) A module P is called *projective* if for every *surjective* homomorphism $f : N \to M$ and every homomorphism $f : P \to M$ there exists a homomorphism $\tilde{f} : P \to N$ such that $f \circ \tilde{f} = g$. (Note: $\tilde{f}$ is not required to be unique – this is not a universal property). (Draw a diagram illustrating this definition). Prove that:
   (a) A module P is projective if and only if it is a direct summand of a free module, i.e., there exists a free module F and a module N such that F is isomorphic to $P \oplus N$.
   (b) Prove that a projective module is flat. (Hint: use Problem 2 above).
   (c) Prove that P is projective if and only if the functor $\mathrm{Hom}(P, -)$ is exact.
(6) Prove that $\mathbb{Q}$ as a $\mathbb{Z}$-module is flat but not projective.
(7) Prove that if A is a principal ideal domain, then any projective A-module is free.
(8) Exercise 5 on p. 44 of AM.
(9) Exercise 12 on p. 45, Parts (iii) and (iv).
(10) Exercise 22 on p. 47 (you will need to understand Exercise 21 for this, but you do not need to hand in Exercise 21).