Problem Set 4. Due Thursday March 16 (or a bit later).

Here AM= Atiyah-Macdonald. and "Singular" is "Singular Introduction to Commutative Algebra" (available online through the library).

(1) Exercise 1 on p.67 of AM.
(2) Exercise 3 on p.67 of AM.
(3) Exercise 4 on p.67 of AM. Here, think of why doesn't this contradict Proposition 5.6?
(4) Exercise 10 on p.67 Part (i) **and** Chapter 6, Exercise 11 of AM. (Note: please also look at Part (ii) but you do not have to write it up and hand in, since we have not covered direct limits).
(5) Exercise 3.1.7 on p. 198 of "Singular" (I think Part (1) has a typo: they mean $\varphi^\#$ not φ^*).
(6) Find the normalization (i.e. the variety of the form $\mathrm{Spec}\, B$ where B is the integral closure of A in its field of fractions, where $X = \mathrm{Spec}(A)$) of the varieties:
  (a) X is a *triple point*: the curve defined by the equation $y^3 = x^3 + x^4$ (hint: divide by x^3 to find an element of the field of fractions that is integral over A, as we did in class).
  (b) A *tacnode*: X is given by the equation $y^2 = x^4 + x^5$ (Hint: $t = y/x$ does not give everything).
(7) Exercise 3.6.5 on p.230 of "Singular".

Recommended but not for handing in: Exercise 11 on p.67 and Exercises 16, 18 from Chapter 3 that it relies on.