Lie Groups. Problem Set 1. Due Friday September 14.

1. Find all Lie subgroups of the additive Lie group K.

2. (a) Show that the real Lie group of orthogonal $n \times n$-matrices $O_n(\mathbb{R})$ is compact. (Hint: use the algebraic equations defining this group.)
   (b) Show that the unitary group U_n of complex unitary $n \times n$-matrices is a *real* Lie group, and it is compact.

3. Show that $GL_n(K)$ acts on the Grassmannian of k-dimensional subspaces of K^n.

4. Let $\mathbb{H}$ be the quaternions (a real division algebra). Show that the group $GL_n(\mathbb{H})$ with the differential (real) manifold structure it gets as an open subset of the real vector space of all $n \times n$-matrices over the quaternions, is a real Lie group of dimension $4n^2$.

5. Let G be a Lie group, $g \in G$. Show that the centralizer $Z(g) = \{h \in G \mid hg = gh\}$ of the element g is a Lie subgroup.