Lie Groups. Problem Set 5. Due Friday November 2.

1. Write down a map of rings that gives the multiplication map on the $\mathbb{C}$-points of $\mathbb{G}_m$.

2. Let $g = \left[\begin{smallmatrix} 1 & i \\ 0 & 1 \end{smallmatrix}\right] \in \mathfrak{sl}_2(\mathbb{C})$. Write down the map of the coordinate rings that gives the left translation by g on $\mathfrak{sl}_2(\mathbb{C})$.

3. Prove that $\{\exp(t \left[\begin{smallmatrix} 1 & i \\ 0 & 1 \end{smallmatrix}\right]) \mid t \in \mathbb{C}\}$ is a Lie subgroup of $\mathrm{GL}_2(\mathbb{C})$ but not an algebraic subgroup. (Hint: you can use the hint from the textbook. It is Problem 7 in Chapter 3, Section 1).

4. Let M be an algebraic variety embedded in a complex affine space, and let $\bar{M}$ be its complex conjugate. Then there is an isomorphism $M^{\mathbb{R}}(\mathbb{C}) \cong M \times \bar{M}$, and every point $x \in M^{\mathbb{R}}(\mathbb{C})$ corresponds to the point $(x, \bar{x}) \in M \times \bar{M}$. (This is exercise 17 in Chapter 2, Section 3).

5. Write down explicitly the coordinate ring of SO_2 (over $\mathbb{C}$). Write down two different automorphisms of order 2 of this ring, and then for each automorphism, write the equations defining the affine variety (over $\mathbb{R}$) whose $\mathbb{R}$-points coincide with the set of fixed points of this automorphism.