Lie Groups. Problem Set 6. Due Monday November 19.

1. Find an example of an irreducible algebraic group such that the set of its real points is not connected.
2. Prove that there are no nontrivial homomorphisms from a quasitorus to the additive group $\mathbb{G}_a$ ($\mathbb{G}_a$ is the notation for the algebraic group s.t. $\mathbb{G}_a(K) = K$).
3. Show that the maps exp and log between the groups of nilpotent (respectively, unipotent) operators are morphisms that are inverses of each other. (Hint: work with the formal power series).
4. Let G be an algebraic group, let $g \in G(\mathbb{C})$. Show that if g^m is semisimple for some positive integer m, then g is semisimple.
5. Show that the intersection of the kernels of all characters of an algebraic group G is a normal algebraic subgroup, and the quotient of G by this subgoup is a quasitorus.
6. Let T_1, T_2 be algebraic tori. Then there is one-to-one correspondence between $\mathrm{Hom}(T_1, T_2)$ and $\mathrm{Hom}(\mathcal{X}(T_2), \mathcal{X}(T_1))$.
7. Show that any subgroup of an irreducible solvable algebraic group G that consists only of semisimple elements has to be commutative. In particular, any finite subgroup of such G is commutative.