Math 534. Written problems, set 2. Due Tuesday, October 11.

(1) Let $\mathfrak{g}$ be a Lie algebra, and let V and W be $\mathfrak{g}$-modules. Let $\mathrm{Hom}(V, W)$ be the space of all linear operators from V to W. Define the action of $\mathfrak{g}$ on $\mathrm{Hom}(V, W)$ by:

$$(X \cdot f)(v) = X \cdot f(v) - f(X \cdot v), \quad X \in \mathfrak{g}, f \in \mathrm{Hom}(V, W).$$

   (a) Show that this makes $\mathrm{Hom}(V, W)$ an $\mathfrak{g}$-module.
   (b) Show that $\mathrm{Hom}(V, W)$ is isomorphic to $V^* \otimes W$.

(2) Humphreys, Exercise 6 on p.24.

(3) Humphreys, Exercise 2 on p.34.