

Math 534. Written problems, set 4. Due December 5.

- (1) Let V be the standard (3-dimensional) representation of $\mathfrak{sl}_3(\mathbb{C})$.
 - (a) Draw the weight diagram of V .
 - (b) Draw the weight diagram for $\text{Sym}^2 V$ (with multiplicities).
 - (c) Prove that the representation $\text{Sym}^n V$ is irreducible for all n .

- (2) This problem is about representations of $\mathfrak{sp}_4(\mathbb{C})$.
 - (a) Draw the root lattice and the weight lattice for C_2 on the same diagram.
 - (b) Draw the weight diagram for the irreducible representation (call it W) of $\mathfrak{sp}_4(\mathbb{C})$ with highest weight $\alpha + \beta$ (where $\{\alpha, \beta\}$ is the standard base for C_2).
 - (c) Let V be the standard (4-dimensional) representation of \mathfrak{sp}_4 . Show that the alternating square $\wedge^2 V$ is the direct sum of W from part (b), and the trivial representation.
 - (d) Show that $\text{Sym}^2 V$ is isomorphic to the adjoint representation.