Math 534. Written problems, set 1. Due Tuesday, September 22.

(1) Let $\mathfrak{g}$ be a Lie algebra.
 (a) Show that if $\mathfrak{h}$ is an ideal in $\mathfrak{g}$, then $[\mathfrak{h}, \mathfrak{h}]$ is also an ideal in $\mathfrak{g}$.
 (b) Let $\mathcal{D}^k\mathfrak{g}$ be the k-th term of the derived series of $\mathfrak{g}$. Show that it is, in fact, an ideal in $\mathfrak{g}$.
 (c) Show that $\mathfrak{g}$ is semisimple if and only if it has no non-trivial *abelian* ideals.

(2) Humphreys, Exercise 3 on p.5.

(3) Humphreys, Exercise 6 on p.5.

(4) **Heisenberg Lie algebra.** Let V be a 3-dimensional vector space over a field F with basis $\{x, y, z\}$. Define the Lie bracket by $[x, y] = z$ and declaring that z is central.
 (a) Show that this defines a Lie algebra, isomorphic to the space of matrices over F of the form $\left(\begin{smallmatrix} 0 & a & c \\ 0 & 0 & b \\ 0 & 0 & 0 \end{smallmatrix}\right)$ with the Lie bracket defined by the usual matrix commutator.
 (b) Show that this is a 2-step nilpotent Lie algebra (i.e. the second term of the lower central series vanishes).