

**Math 534. Written problems, set 3. Due December 19.**

- (1) Let  $V$  be the standard (3-dimensional) representation of  $\mathfrak{sl}_3(\mathbb{C})$ .
  - (a) Draw the weight diagram of  $V$ .
  - (b) Draw the weight diagram for  $\text{Sym}^2 V$  (with multiplicities). Is this representation irreducible?
  - (c) Draw the weight diagram for  $\wedge^2 V$  (with multiplicities). Is this representation irreducible?
  
- (2) This problem is about representations of  $\mathfrak{sp}_4(\mathbb{C})$ .
  - (a) Draw the root lattice and the weight lattice for  $C_2$  on the same diagram.
  - (b) Draw the weight diagram for the irreducible representation (call it  $W$ ) of  $\mathfrak{sp}_4(\mathbb{C})$  with highest weight  $\alpha + \beta$  (where  $\{\alpha, \beta\}$  is the standard base for  $C_2$ ).
  - (c) Let  $V$  be the standard (4-dimensional) representation of  $\mathfrak{sp}_4$ . Show that the alternating square  $\wedge^2 V$  is the direct sum of  $W$  from part (b), and the trivial representation.