

**Math 534. Written problems, set 2.**

- (1) Let  $\mathfrak{g}$  be a Lie algebra, and let  $V$  and  $W$  be  $\mathfrak{g}$ -modules. Let  $\text{Hom}(V, W)$  be the space of all linear operators from  $V$  to  $W$ . Define the action of  $\mathfrak{g}$  on  $\text{Hom}(V, W)$  by:

$$(X \cdot f)(v) = X \cdot f(v) - f(X \cdot v), \quad X \in \mathfrak{g}, f \in \text{Hom}(V, W).$$

- (a) Show that this makes  $\text{Hom}(V, W)$  an  $\mathfrak{g}$ -module.  
(b) Show that  $\text{Hom}(V, W)$  is isomorphic to  $V^* \otimes W$ .
- (2) Humphreys, Exercise 5 on p.21.  
(3) Humphreys, Exercise 6 on p.24.