Math 534. Written problems, set 4. Due December 23.

Only the parts without the asterisk are mandatory. The rest are optional and can be done in lieu of the presentation.

(1) Let V be the standard (3-dimensional) representation of $\mathfrak{sl}_3(\mathbb{C})$.
   (a) Draw the weight diagram of V (with multiplicities).
   (b) Draw the weight diagram for $\mathrm{Sym}^2 V$ (with multiplicities).
   (c)* Prove that the representation $\mathrm{Sym}^n V$ is irreducible for all n.

(2) This problem is about representations of $\mathfrak{sp}_4(\mathbb{C})$.
   (a) Draw the root lattice and the weight lattice for C_2 on the same diagram.
   (b) Draw the weight diagram for the irreducible representation (call it W) of $\mathfrak{sp}_4(\mathbb{C})$ with highest weight $\alpha + \beta$ (where $\{\alpha, \beta\}$ is the standard base for C_2).
   (c)* Let V be the standard (4-dimensional) representation of $\mathfrak{sp}_4$. Show that the alternating square $\wedge^2 V$ is the direct sum of W from part (b), and the trivial representation.
   (d)* Show that $\mathrm{Sym}^2 V$ is isomorphic to the adjoint representation.