Acknowledgement

UBC's Point Grey Campus is located on the traditional, ancestral, and unceded territory of the xʷməθkʷəy̓əm (Musqueam) people. The land it is situated on has always been a place of learning for the Musqueam people, who for millennia have passed on in their culture, history, and traditions from one generation to the next on this site.

Course Information

Course Title	Course Code Number	Credit Value
Lie Theory I – Lie algebras	Math 534	3

Textbook:

- James E. Humphreys, "Introduction to Lie algebras and representation theory".
- W. Fulton and J. Harris "Representation theory. A first course" will also be useful (you do not have to buy it).

Prerequisites

Good understanding of linear algebra, including Jordan canonical form, and a first course in group theory.

Contact

Course Instructor(s)	Contact Details	Office Location	Office Hours
Julia Gordon	email: `gor "at" math.ubc.ca`. (you can expect a response within 48 hours). You can also just catch me after class.	MATH 217	By appointment – please email me to set a time

Website

Instructor's home page: `www.math.ubc.ca/~gor` ;
Follow the link for Math 534 under "Teaching this term". The most current course information will be posted on the course website.

Course description

Lie theory studies Lie groups, which are manifolds with group structure. These objects are important in various areas of mathematics, from PDE to number theory, as well as in physics. This is a very first course in Lie theory, and it will focus entirely on the study of Lie algebras. Lie algebras are vector spaces with an additional algebraic structure, called "Lie bracket". They arise as tangent spaces to Lie groups.

Except for this introduction, Lie groups will not be mentioned in this course (unless someone chooses to do a presentation about them in the end), hence no familiarity with the notion of a manifold will be required. Instead, we will focus entirely on the study of Lie algebras and their representations, by purely algebraic methods. It turns out that they have a beautiful classification. We will define and classify the root systems; talk about weights, highest weight modules, universal enveloping algebra, and ultimately will prove the classification theorem for Lie algebras over an algebraically closed field of characteristic zero (we can just think of the field of complex numbers).

We will cover most of the textbook (except maybe the last chapter), plus some examples (for the examples, the book by Fulton and Harris might be useful).

At the end of the class, we will discuss some topics depending on the participants' interests (Lie groups, introduction to algebraic groups, some in-depth topics related to representation theory...)

Approximate schedule of Topics

Weeks 1-3	Lie algebras: an overview; the basic definitions. The 2-dimensional Lie algebras; adjoint representation; proof that sl(2) is simple. The radical. Nilpotent and solvable Lie algebras. Engel's Theorem, Lie's theorem. Cartan's criterion.
Weeks 4-5	Jordan-Chevalley decomposition. Complete reducibility of representations: the Killing form, Casimir element, Weyl's theorem. Representations of sl(2). Root space decomposition.
Weeks 6-7	Root systems; the definition of the Dynkin diagram. Bases, action of the Weyl group; Automorphisms of root systems. Construction of the root system from simple roots. The classification theorem for the semisimple Lie algebras.
Weeks 8-9	Automorphisms of Lie algebras, Cartan subalgebras, inner automorphisms. Borel subalgebras; conjugacy of Borel subalgebras. The universal enveloping algebra, Poincaré-Birkhoff-Witt theorem. Sketch of Existence theorem, Serre's relations.
Weeks 10-11	Representation theory: the weight lattice; weight spaces; standard cyclic modules; classification of the finite-dimensional representations.
Weeks 12-13	Presentations on various topics, including Freudenthal's, Weyl's and Kostant's character formulas.

Learning Activities and Assessments of Learning

Everyone will be expected to do either an in-class presentation during the last two weeks of class or a short essay (in place of the final exam). Possible presentation topics and resources will be decided on during the first few weeks of class (depending on everyone's interests). There will be some written homework problems, due roughly once in two weeks, but most homework will be discussed in class (i.e., volunteers will present homework solutions with discussion). The in-class homework discussions will happen approximately once in two weeks, the dates will be posted on the course website in advance.

The mark will be based on your participation in the homework discussions, the written homework, and your final presentation, according to the following standard criteria:

- An A+ (90% and above) represents excellent work. A student receiving an A+ is encouraged to pursue research in the area of the course. This means, doing most homework problems, active participation in the homework discussions, and an excellent final presentation.
- An A (85% to 89%) represents good work. A student receiving an A is prepared to take more advanced courses in the area and is encouraged to consider research in the area of the course.
- An A- (80% to 84%) represents satisfactory work – in this course, this means doing some but not all of the homework problems, participating somewhat in the discussions, and the final presentation that has some flaws.
- Any grade of 79% or below (B+ or below) represents unsatisfactory work. A student receiving a B+ or below will probably need to improve their understanding in order to take additional courses in the area. An instructor assigning a B+ or below deems that the student did not fulfill the department's expectations for a graduate student in that course. **Grades below 80% in this course will not be assigned without warning – if you feel you are not keeping up for any reason, please come talk to me; if I feel you are not keeping up, I will initiate the conversation.**

University Policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the `https://senate.ubc.ca/policies-resources-support-student-success`UBC Senate website.