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Replicator dynamics of reward & reputation in public goods games

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ABSTRACT

Public goods games have become the mathematical metaphor for game theoretical investigations of cooperative behavior in groups of interacting individuals. Cooperation is a conundrum because cooperators make a sacrifice to benefit others at some cost to themselves. Exploiters or defectors reap the benefits and forgo costs. Despite the fact that groups of cooperators outperform groups of defectors, Darwinian selection or utilitarian principles based on rational choice should favor defectors. In order to overcome this social dilemma, much effort has been expended for investigations pertaining to punishment and sanctioning measures against defectors. Interestingly, the complementary approach to create positive incentives and to reward cooperation has received considerably less attention—despite being heavily advocated in education and social sciences for increasing productivity or preventing conflicts. Here we show that rewards can indeed stimulate cooperation in interaction groups of arbitrary size but, in contrast to punishment, fail to stabilize it. In both cases, however, reputation is essential. The combination of reward and reputation result in complex dynamics dominated by unpredictable oscillations.

1. Introduction

The evolution and maintenance of cooperation marks a central topic across behavioral disciplines ranging from microbial populations to human societies. Over recent years, altruistic punishment as an efficient way to encourage and stabilize cooperation has attracted considerable attention from theorists as well as experimentalists (Clutton-Brock and Parker, 1995; Fehr and Gächter, 2002; Hauert et al., 2007; Sigmund, 2007; Rockenbach and Milinski, 2006; Gürerk et al., 2006; Henrich and Boyd, 2001; Gächter et al., 2008). Colman (2006) summarizes the progress by quipping that "we seem to have replaced the problem of explaining cooperation with that of explaining altruistic punishment". Interestingly, the complementary approach – rewarding good behavior rather than punishing bad behavior – received much less attention. Although the importance of selective incentives has long been recognized in the social sciences (Oliver, 1980). For a recent survey of theoretical advances and behavioral experiments involving positive and negative incentives to cooperate, see Sigmund (2007). Only more recent studies proposing alternatives to punishment seem to gain momentum (Andreoni et al., 2003; Dreber et al., 2008; Rand et al., 2009; Sefton et al., 2007; DeSilva and Sigmund, 2009; Hilbe and Sigmund, 2010).

For pairwise interactions in the prisoner's dilemma it has been demonstrated that adding the opportunity to reward other cooperators may lead to unpredictable oscillatory dynamics quite in contrast to the complementary approach to allow the punishment of cheaters (Sigmund et al., 2001). Most importantly, punishment can have a stabilizing effect on cooperation (or any other behavior that evades punishment) (Boyd and Richerson, 1992), whereas rewarding may encourage cooperation but fails to stabilize it.

For the successful emergence of cooperative behavior cooperators first need to gain a foothold in the population and, once established, cooperation needs to be maintained. These are two rather different and largely independent challenges. Punishment is good at stabilizing cooperation because if everybody cooperates little or no costs arise from the need to punish cheaters. Conversely, in a population of cheaters punishing left and right is extremely expensive and therefore punishers perform poorly and fail to establish themselves in the population. With rewards it is just the opposite: rewarding cooperation is expensive in a population of cooperators but cheap if everyone defects. Hence rewards can inspire cooperation but once cooperation is established those that cooperate but do not provide rewards (second order exploiters) are better off and thwart the attempts to sustain cooperation based on rewards.

Here we extend the work by Sigmund et al. (2001) and model the effects of rewarding in public goods interactions in groups of arbitrary size N. Interactions occur in two stages: in the first stage, all individuals engage in a traditional public goods game, where each player may invest a certain amount c into a common pool, knowing that the total contributions will be multiplied by $r>1$ and then equally divided among all N members of the

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group—irrespective of whether they contributed or not. Thus, the public good provides a benefit $B=n_c\ c\ r/N$ to everyone, where n_c denotes the number of cooperators among the $N-1$ co-players. Contributors incur net costs of $C=(1-r/N)c$, which takes into account that part of the investment returns to the contributor. In the following we always assume $r<N$ such that each invested dollar returns less to the investor. Selfish strategies maximize (short-term) benefits by cutting costs and hence do not invest in the common pool. Thus, a group of selfish players will not increase their initial capital—however, had everybody contributed, everybody would have profited and received a net benefit of $(r-1)c$. The resulting conflict of interest between the individual and the group is termed a social dilemma (Dawes, 1980; Hauert et al., 2006).

In the second stage each individual may choose to reward those that contributed to the public good in an attempt to encourage and maintain cooperation. Rewarding another individual incurs costs γ and the beneficiary receives β with $\beta\geq\gamma$, with equality corresponding to a simple payback. As before, rational players attempt to minimize their costs and hence choose not to reward—but then the incentive to cooperate in the first stage disappears. Consequently, the selfish strategy is to neither contribute nor reward.

Thus, this two stage interaction admits four strategic types: (i) the negligent cooperators that contribute to the public good but do not reward other contributors; (ii) the cheaters, which neither contribute to the public good nor reward others; (iii) the prosocial individuals, which both contribute to the public good and reward other contributors; and finally (iv) the cautious individuals that refrain from risky contributions to the public good but do reward other contributors.

The above conclusion that selfish individuals cheat is implicitly built on the assumption that interactions are fully anonymous and not repeated. While these conditions may be satisfied in experimental settings, they rarely apply in natural interactions. Instead, in most cases interactants will have some expectations concerning the behavioral types of their partners—either based on experiences in previous encounters, through observations of third party interactions or through communication and gossip. This kind of reputation is crucial in order to establish cooperation based on indirect reciprocity in theory (Nowak and Sigmund, 1998, 2005) as well as experiments (Wedekind and Milinski, 2000; Milinski et al., 2002). In the present context this means that all individuals carry a reputation, which may become known to their co-players. More specifically, with a small probability, ν, individuals may learn, whether their partners reward contributions to the public good or not, and they may adjust their behavior accordingly.

In the following we consider the scenario where non-contributors reconsider their behavior and contribute whenever they know that *all* of their co-players reward contributors. This is actually a selfish behavioral response if the accumulated rewards exceed the net costs of contributions, $(N-1)\beta>(1-r/N)c$. In the following we assume that this weak constraint is always satisfied. In principle, the number of rewarders required to induce contributions could be lowered to any threshold but, to simplify the analysis, we chose the most stringent case where the rewards of all $N-1$ co-players are needed to change the mind of non-contributors. Therefore, all but one rewarders in an interaction group can convince a non-contributing individual to reconsider its behavior and induce cooperative contributions. This precludes simple conclusions concerning selfish actions because the short-term benefits now depend on the composition of the interaction group.

2. Model

The evolutionary success of the different strategies can be analyzed by considering a large population where everyone interacts with every other member of the population with equal probability, i.e. binomial sampling of interaction groups. The state of the population is then fully determined by the frequencies of the four strategic types: the negligent cooperators, x, the cheaters, y, the prosocial individuals, v, and the cautious ones, w, with $x+y+v+w=1$. In this case, evolutionary changes of the population can be modeled by the replicator dynamics (Hofbauer and Sigmund, 1998). The replicator dynamics simply states that strategies, which perform better than the population on average, increase in abundance at the cost of those that perform worse:

$$\dot{x}=x(P_x-\overline{P}),\quad \dot{y}=y(P_y-\overline{P}),\quad \dot{v}=v(P_v-\overline{P}),\quad \dot{w}=w(P_w-\overline{P}), \tag{1}$$

where P_i indicates the average payoff of strategic type i and $\overline{P}=xP_x+yP_y+vP_v+wP_w$ denotes the average population payoff. If interaction groups are randomly formed according to binomial sampling then the probability that a focal individual interacts with n_i individuals of type i among its $N-1$ co-players is given by

$$\frac{(N-1)!}{n_x!n_y!n_v!n_w!}x^{n_x}y^{n_y}v^{n_v}w^{n_w}, \tag{2}$$

with $n_x+n_y+n_v+n_w=N-1$. Thus, e.g. the average number of contributors among the focal individuals' co-players amounts to

$$\sum_{s=0}^{N-1}\frac{(N-1)!}{s!(N-1-s)!}(x+v)^s(y+w)^{N-1-s}=(x+v)(N-1). \tag{3}$$

Similarly, on average, each individual faces $(v+w)(N-1)$ rewarders etc. This yields the following average payoffs for each strategic type:

$$P_x=B-C+(v+w)(N-1)\beta, \tag{4a}$$

$$P_y=B+v(v+w)^{N-1}\left[\left(\frac{r}{N}-1\right)c+(N-1)\beta\right], \tag{4b}$$

$$P_v=B-C+(N-1)((v+w)\beta-(x+v)\gamma)+v(y+w)(v+w)^{N-2}(N-1)\left(\frac{r}{N}c-\gamma\right), \tag{4c}$$

$$P_w=B-(x+v)(N-1)\gamma+v(v+w)^{N-2}\left[\left(\left(\frac{r}{N}-1\right)c+(N-1)\beta\right)(v+w)+(y+w)(N-1)\left(\frac{r}{N}c-\gamma\right)\right], \tag{4d}$$

where B and C refer to the benefits and costs of the public goods interaction given by

$$B=(x+v)(N-1)\frac{r}{N}c, \tag{4e}$$

$$C=\left(1-\frac{r}{N}\right)c. \tag{4f}$$

Because of the normalization, $x+y+v+w=1$, the dynamics unfolds in the state space spanned by the simplex S_4. In addition, note that $P_x+P_w=P_y+P_v$ always holds and therefore the replicator dynamics admits a constant of motion given by $xw/yv=K$, where K is a positive constant (Sigmund et al., 2001). Consequently, the simplex S_4 is foliated into invariant manifolds W_K that correspond to saddle-like surfaces spanned by the frame $X-Y-W-V-X$, where X, Y, V, W denote the corners of S_4 with $x=1$, $y=1$, $v=1$ and $w=1$, respectively (see Fig. 1).

Along the boundary of the manifolds W_K the dynamics (i) points from $X\rightarrow Y$, which reflects the fact that in the absence of reward, defectors always outperform cooperators; (ii) is neutral along $Y-W$, represented by a line of fixed points, in the absence of reputation, $\nu=0$, because no one ever cooperates or rewards and hence $P_y=P_w=0$ (see Fig. 1a). However, for $\nu>0$ the dynamics

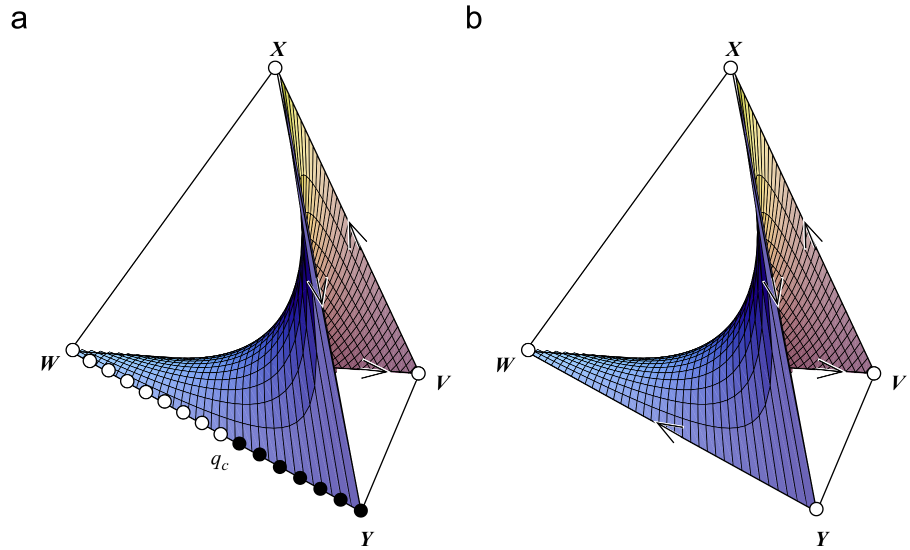

Fig. 1. Invariant manifold W_K embedded in the simplex S_4 with the dynamics along the boundary of W_K. The four corners X, Y, V, W denote the four homogenous states with $x=1$, $y=1$, $v=1$ or $w=1$. (a) For $v=0$ the dynamics along the YW-edge is neutral and consists of a line of fixed points because in the absence of reputation no one ever cooperates and it is impossible to discriminate between pure defectors, y, and those that would reward cooperation, w. The point q_c divides the YW-edge into stable (q_cY, marked by filled circles) and unstable (q_cW, marked by open circles) segments. (b) For $v>0$ the degeneracy of the YW-edge is resolved such that $Y\to W$ (provided that $(N-1)\beta>(1-r/N)c$ and $rc/N>\gamma$) and the boundary of W_K represents a heteroclinic cycle.

points from $Y\to W$ because the risk averse types occasionally manage to induce cooperation, which requires that the total reward exceeds the net costs of cooperation $(N-1)\beta>(1-r/N)c$, and hence $P_w>P_y$, provided that the costs incurring from rewards are less than the benefits from one contributor, $\gamma<rc/N$. The dynamics then (iii) points from $W\to V$ because if everybody rewards, cooperation pays off and finally (iv) from $V\to X$ because negligent cooperators save the costs of rewarding. This closes the circle. Hence, for $v>0$ the boundary of W_K represents a heteroclinic cycle (see Fig. 1b).

Analyzing the dynamics of the full system requires first to locate potential fixed points in the interior of W_K and determine their stability. The dynamics on each manifold is determined by two dynamical variables. For simplicity and convenience, we group the contributors, $p=x+v$, and the rewarders, $q=v+w$. The dynamical equations then become

$$\dot{p}=p(P_p-\overline{P}),\quad \dot{q}=q(P_q-\overline{P}), \tag{5}$$

where $P_p=(x/p)P_x+(v/p)P_v, P_q=(v/q)P_v+(w/q)P_w$ denote the average payoffs to contributors and rewarders, respectively, and $\overline{P}$ indicates the (unchanged) average population payoff. Some algebra then yields

$$P_p=\left(\frac{r}{N}-1\right)c+(N-1)\left[\frac{r}{N}cp+\beta q+\frac{v}{p}\left(vq^{N-2}(1-p)\left(\frac{r}{N}c-\gamma\right)-p\gamma\right)\right], \tag{6a}$$

$$P_q=\ (N-1)p\left(\frac{r}{N}c-\gamma\right)+\frac{v}{q}\left[\left(\frac{r}{N}-1\right)c(1-vq^{N-1})+(N-1)q\beta(1-vq^{N-2})\right]$$
$$+vq^{N-2}\left[q\left(\left(\frac{r}{N}-1\right)c+(N-1)\beta\right)+(N-1)(1-p)\left(\frac{r}{N}c-\gamma\right)\right], \tag{6b}$$

$$\overline{P}=p((r-1)c+q(N-1)(\beta-\gamma))+v(1-p)q^{N-1}((r-1)c+(N-1)(\beta-\gamma)). \tag{6c}$$

Note that for clarity, the expressions for P_p and P_q in Eq. (6) still depend on the frequency of the prosocial strategy v. Therefore, to complete the transformation v needs to be expressed in terms of p and q:

$$v=\begin{cases}\dfrac{(K-1)(p+q)-K+\sqrt{4(K-1)pq+((K-1)(p+q)-K)^2}}{2(K-1)} & K\neq 1,\\ pq & K=1.\end{cases} \tag{7}$$

Note that v is continuous in K and hence the limit $K\to 1$ is unique and well defined. The fixed points of the dynamics satisfy $\dot{p}=0$ and $\dot{q}=0$. This trivially holds for $p=0$, 1 and $q=0$, 1. The four possible combinations correspond to the four homogenous states X, Y, V, W marking the corners of S_4. In addition, non-trivial interior fixed points may exist. Unfortunately, however, general solutions of $\dot{p}=0, \dot{q}=0$ are analytically inaccessible for $N>2$.

2.1. Dynamics on W_1

For $K=1$ and hence $v=pq$ the dynamics can be fully analyzed. Setting $\dot{q}=0$ (see Eq. (5)) specifies the equilibrium fraction of contributors, $\hat{p}$, at a fixed point **Q** in terms of the equilibrium fraction of rewarders, $\hat{q}$:

$$\hat{p}=1-\frac{1}{1-v\hat{q}^{N-2}\left(1-\dfrac{r}{N}\dfrac{c}{\gamma}\right)}. \tag{8}$$

Solving $\dot{p}=0$ and inserting $\hat{p}$ returns $\hat{q}$ as the solution of $F(\hat{q})=0$ with

$$F(q)=\left(1-\frac{r}{N}\right)c(1-vq^{N-1})-(N-1)\beta q(1-vq^{N-2}). \tag{9}$$

It follows that **Q** is unique (if it exists) because $F(q)$ has a unique root in (0,1). In order to verify this, note that $F(0)>0$, $F(1)<0$ (provided that $(N-1)\beta>(1-r/N)c$, i.e. the maximum reward exceeds the net costs of contributions to the public good) and because the curvature of $F(q)$ is always positive, i.e. $F''(q)>0$.

In the absence of reputation ($v=0$), Eq. (8) indicates that the interior fixed point **Q** does not exist (see Fig. 2). In fact, in the limit $v\to 0$, the system undergoes a complex bifurcation where **Q** is replaced by the line of fixed points along the YW-edge ($p=0$). The point $q_c=(1-r/N)c/((N-1)\beta)$ divides the line into two segments: for $q<q_c$ (along the Yq_c-segment) the fixed points are stable and

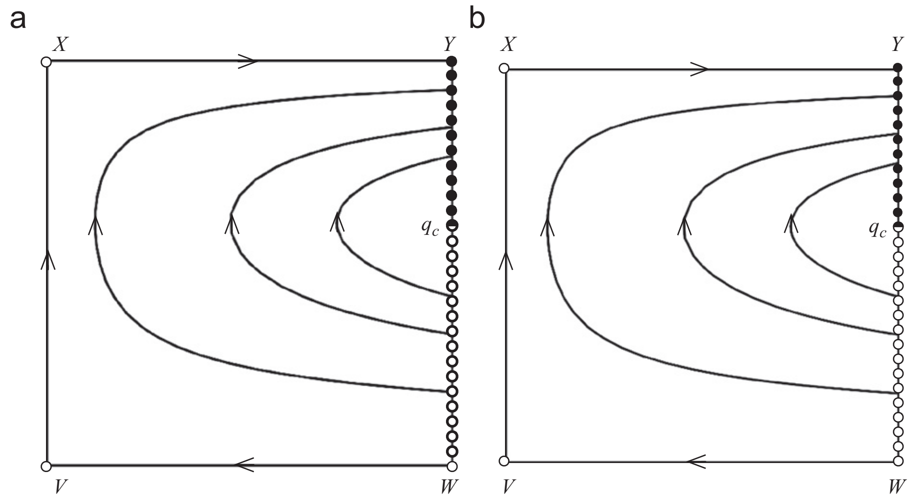

Fig. 2. Dynamics of public goods interactions with reward but without reputation on the invariant manifold W_1 (see Fig. 1a). The two panels show the dynamics for (a) pairwise interactions, $N=2$, and (b) interactions in groups of $N=5$. In order to simplify comparisons of the dynamics for different interaction group size N, the parameters are chosen such that effective costs of cooperation $(1-r/N)c$ as well as the maximum total costs, $(N-1)\gamma$, and benefits, $(N-1)\beta$, of rewarding are equal. The main difference concerns the returns for mutual cooperation, $(r-1)c$, which increases with N. For $v=0$ this results in essentially indistinguishable dynamics as illustrated here for $N=2$ and $N=5$. Parameters: (a) $N=2$, $r=1.2$, $c=1$, $\beta=1$, $\gamma=0.2$ and (b) $N=5$, $r=3$, $c=1$, $\beta=0.25$, $\gamma=0.05$.

unstable for $q>q_c$ (q_cW-segment). In the long run, the population will thus spend most of the time along the stable Yq_c-segment, i.e. in the vicinity of the asocial cheater state. Random shocks may lead to neutral drift along the YW-edge but whenever contributors appear, the population exhibits a brief burst of cooperation but then ends up closer to the cheater state.

For $v>0$, the interior fixed point $\mathbf{Q}$ exists and for $K=1$ its stability can be determined by the Jacobian matrix at $\mathbf{Q}$:

$$\mathbf{J}=(N-1)\begin{pmatrix} 0 & p(1-p)\left(\beta-vq^{N-2}\left(\left(\frac{r}{N}-1\right)c+(N-1)\beta\right)\right) \\ -q(1-q)\left(\gamma+vq^{N-2}\left(\frac{r}{N}c-\gamma\right)\right) & (N-1)(N-2)\gamma p(1-q) \end{pmatrix}. \tag{10}$$

For $N>2$ it immediately follows that $\mathbf{Q}$ is always unstable because $\mathrm{tr}\,\mathbf{J}>0$. More precisely, $\mathbf{Q}$ must be an unstable focus because it is unique and because of the heteroclinic cycle along the boundary of W_1. For $N=2$, $\mathrm{tr}\,\mathbf{J}=0$ and $\mathbf{Q}$ is neutrally stable and surrounded by closed periodic orbits (see Fig. 3 and Sigmund et al., 2001).

2.2. Dynamics on W_K

The analysis of the dynamics for arbitrary K is much harder but also reveals some interesting aspects of the dynamics. Fortunately, it turns out that the coordinates of $\mathbf{Q}$, $\hat{p}$ and $\hat{q}$, do not depend on K. This is readily verified by inserting $\hat{p}$ into the general dynamics of Eq. (5) and noting that both $\dot{p}$ and $\dot{q}$ become proportional to $F(\hat{q})$, which is zero at $\mathbf{Q}$. Unfortunately, however, the stability analysis cannot be easily generalized to arbitrary K except for pairwise interactions with $N=2$ (Sigmund et al., 2001). In this case, $\mathbf{Q}$ is an unstable focus for $K<1$ and a stable focus for $K>1$ (see Fig. 3).

Numerical analysis of the real part of the complex conjugate eigenvalues $\lambda_\pm$ of the Jacobian matrix at $\mathbf{Q}$ suggests that $\mathbf{Q}$ is always unstable for $K\leq 1$. For sufficiently large K and suitable parameters $\mathbf{Q}$ can become stable (see Figs. 4, 5). If $\mathbf{Q}$ changes stability across manifolds, the system undergoes a sub-critical Hopf-bifurcation (Kuznetsov, 2004). For manifolds with $K<K_{\mathrm{Hopf}}$, $\mathbf{Q}$ is unstable and turns into a stable focus surrounded by an unstable limit cycle for $K>K_{\mathrm{Hopf}}$ (see Fig. 5). The limit cycle divides the manifold into two basins of attraction. Depending on the initial configuration the system either converges to the stable fixed point $\mathbf{Q}$ or approaches the heteroclinic cycle. On manifolds with larger K the size of the limit cycle and hence of the basin of attraction of $\mathbf{Q}$ increases. The limit cycle seems to persist and converge to the heteroclinic cycle in the limit $K\to\infty$. This suggests that the boundary of W_K is always attracting for $N>2$. However, note that segments of the limit cycle quickly converge to the boundary (see Fig. 5c, d), which challenges the accuracy of the numerical integration of the dynamics.

Small random shocks can push the state from one manifold to another and hence change the value of K. These changes in K can become large whenever the state is close to the boundary. If $\mathbf{Q}$ changes stability across manifolds, the dynamics can become very complex with alternating periods where the state converges to $\mathbf{Q}$ and periods where the state approaches the heteroclinic cycle.

3. Discussion

Introducing the opportunity to reward contributors has the potential to alleviate the social dilemma posed by communal enterprises. However, this requires that rewards are sufficiently appealing or, more precisely, that the accumulated rewards may exceed the costs of cooperation, as well as that individuals carry a reputation and others may learn whether their interaction partners offer rewards and may adjust their behavior accordingly. Effects of reputation are considered by introducing a small probability, v, that non-contributors may find out whether their co-players offer rewards and if everybody does they are reformed and contribute to the public good. This is actually a selfish choice whenever the rewards offset the costs of cooperation, $(N-1)\beta>(1-r/N)c$, and it is even mutually beneficial if the benefits from gaining one additional contributor exceed the costs of rewarding, $rc/N>\gamma$. If both conditions are met, risk averse individuals that do not contribute to the public good but are willing to provide rewards to those that do, provide an escape hatch out of states of mutual defection. These cautious individuals are capable of undermining asocial types that neither cooperate nor reward while maintaining the ability to inspire cooperative contributions. However, handing out rewards is unable to stabilize cooperation. If everybody cooperates and rewards, individuals face the temptation to further increase their profits by saving the costs of rewarding others. These second

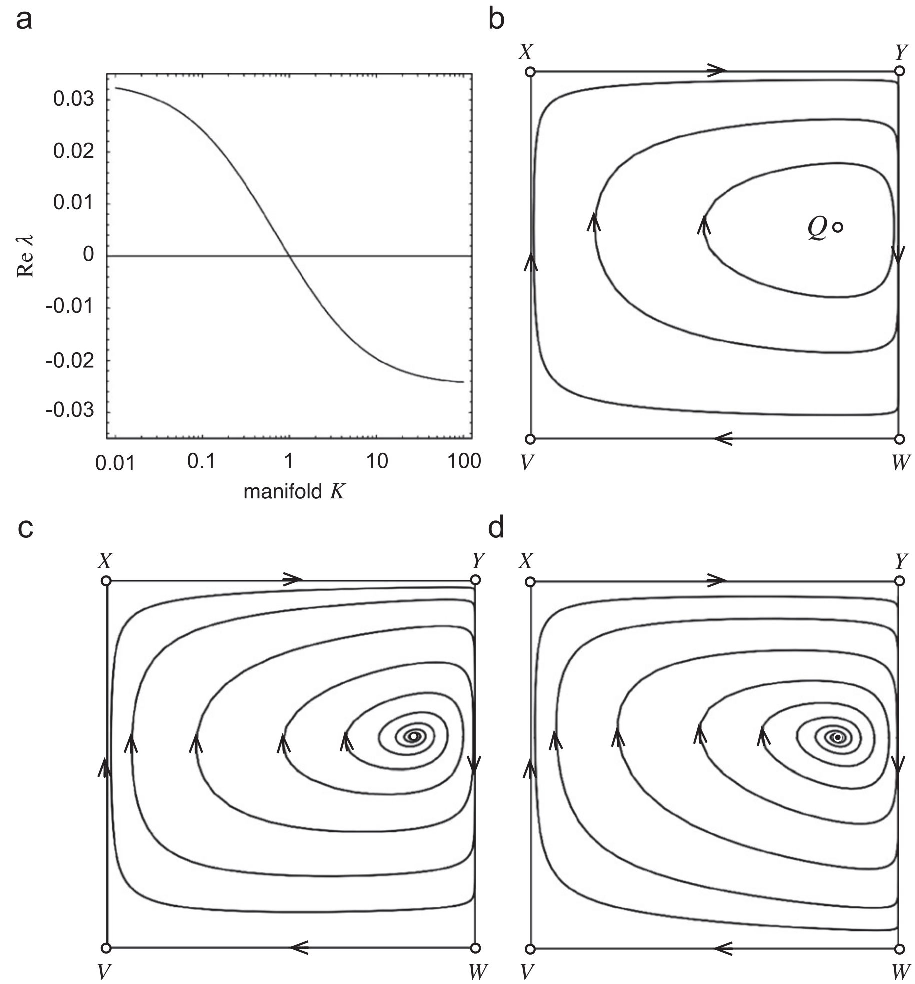

Fig. 3. Dynamics of cooperation, reward and reputation in public goods games for $N=2$ on different manifolds W_K. (a) Depicts the real part of the eigenvalues of the Jacobian matrix (10), Re λ, at **Q** across manifolds. (b) For $K=1$, Re $\lambda=0$ and **Q** is neutrally stable, surrounded by closed orbits. (c) For $K<1$, Re $\lambda>0$ and **Q** is unstable and all orbits converge to the heteroclinic cycle along the boundary of W_K. (d) Conversely, for $K>1$, Re $\lambda<0$ and all orbits converge to the stable focus **Q**. Parameters: $N=2$, $r=1.2$, $c=1$, $\beta=1$, $\gamma=0.2$, $v=0.1$, **c** $K=1/4$, (d) $K=4$.

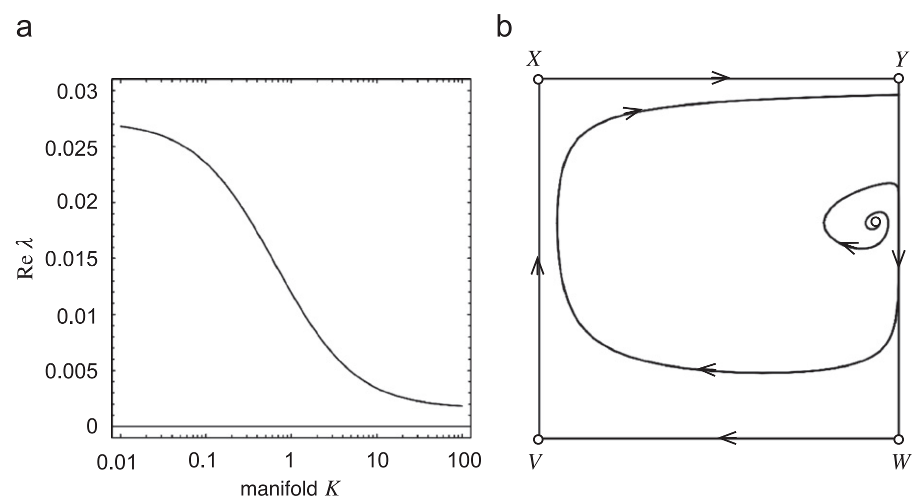

Fig. 4. Dynamics of cooperation, reward and reputation in public goods games for $N=5$ on different manifolds W_K (c.f. Fig. 3). (a) The interior fixed point **Q** represents an unstable focus on all manifolds because Re $\lambda>0$ always holds. (b) All trajectories spiral away from **Q** and converge to the heteroclinic cycle along the boundary of W_1. Parameters: $N=5$, $r=3$, $c=1$, $\beta=0.25$, $\gamma=0.05$, $v=0.1$.

order free-riders pave the way for the successful return of asocial types. Such cycles seem characteristic for cooperation and reward in public goods games.

For the numerical solutions shown here $\gamma<\beta$ holds (see Figs. 2–5), such that the costs of reward are less than its benefits. In the case of rewards such an increase in value could, for

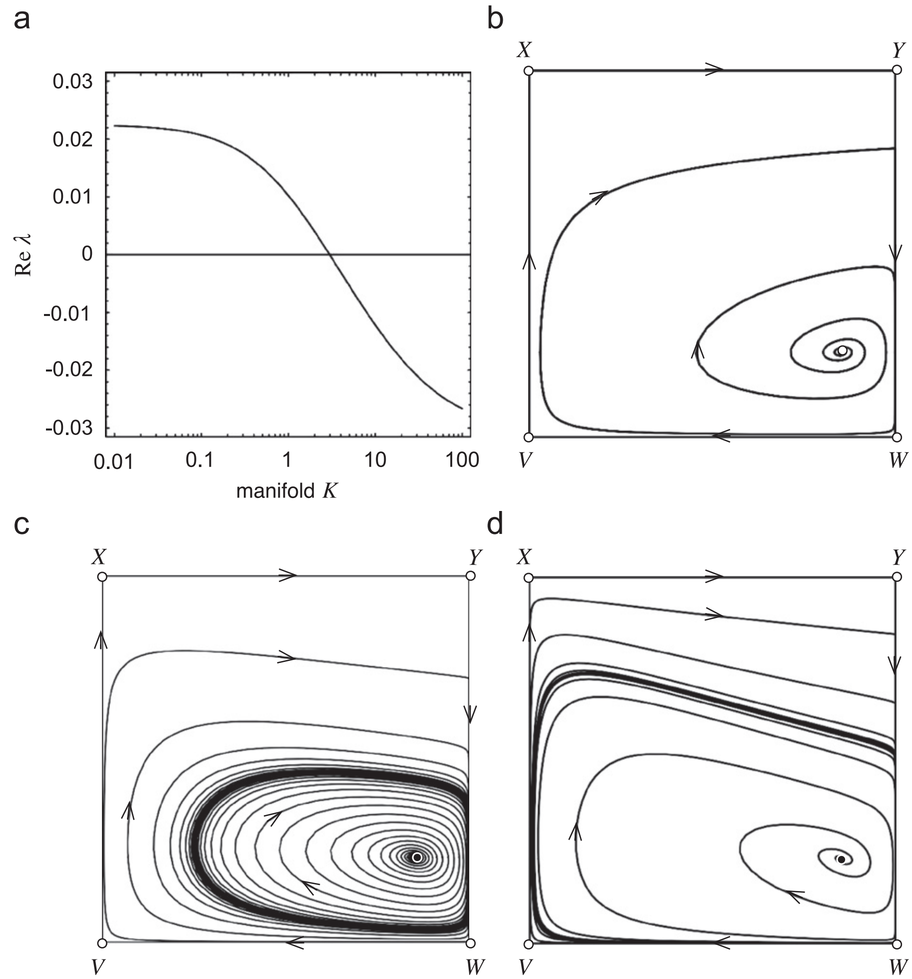

Fig. 5. Hopf-bifurcation and unstable limit cycles in public goods games with reward and reputation. (a) Reducing the return of the public goods interaction as compared to Fig. 4 renders the interior fixed point **Q** stable (Re $\lambda<0$) for sufficiently large K. When moving across the manifolds characterized by K, the system actually undergoes a sub-critical Hopf-bifurcation at $K_{\mathrm{Hopf}}\approx 2.975$. (b) For $K<K_{\mathrm{Hopf}}$ the fixed point **Q** is unstable and all trajectories spiral toward the heteroclinic cycle along the boundary of W_K. (c) For $K>K_{\mathrm{Hopf}}$, **Q** is stable but its basin of attraction is limited by the surrounding unstable limit cycle. Hence the heteroclinic cycle remains an attractor of the dynamics. (d) Increasing K increases the size of the unstable limit cycle and hence the basin of attraction of **Q**. Parameters: $N=5$, $r=1.2$, $c=1$, $\beta=0.25$, $\gamma=0.05$, $v=0.1$; (b) $K=1$; (c) $K=4$; (d) $K=10$.

example, relate to feelings of gratitude, whereas $\gamma=\beta$ would correspond to a simple payback. Obviously greater leverage makes rewards more attractive and creates stronger incentives to use them as a means to encourage cooperation but does not affect the qualitative dynamics of the system. Both situations have been used in experimental settings (Rockenbach and Milinski, 2006; Sefton et al., 2007).

In the experiments by Milinski et al. (2002) individuals had the opportunity to build a reputation in public goods interactions and contributors had a higher chance to receive help in subsequent indirect reciprocity interactions, which can be interpreted as rewarding cooperative behavior.

Note that we could also consider different effects of reputation where, with a small probability, μ, contributors switch to defection if they find out that too few or none of their co-players offers rewards. However, μ has little effects on the dynamics—in particular, μ cannot eliminate the line of fixed points along the YW-edge, which is crucial for the escape from mutual defection (Sigmund et al., 2001).

Traditionally, the complementary approach to punish those that failed to contribute to the public good has attracted considerably more attention. It seems clear that punishment can stabilize cooperation (or anything else) (Boyd and Richerson, 1992), even if it is costly, but it is less clear how social norms based on punishment could get established in a population. If defection abounds, punishers carry the costs of punishing left and right, which results in a poor performance. However, if punishers rule and everybody cooperates, punishment is cheap because only the odd errant defector requires admonition. In the absence of reputation, punishing cooperators can be undermined by second order free-riders that cooperate but are not willing to carry the costs of punishment which provokes the return of defectors (Sigmund et al., 2001; Hauert et al., 2007). The demise of punishment (and cooperation) is prevented by reputation, if contributors may switch to defection whenever they learn, say with a small probability μ, that none of their co-players punishes (Hauert et al., 2004). Thus cooperation (and punishment) is stabilized by a lowering of the morals: cheat whenever you can get away with it. The other effect of reputation, where non-contributors may learn with a small probability v, whether their co-players punish and switch to cooperation if they do, barely affects the dynamics.

Interestingly, the effects of reputation that are crucial to stabilize cooperation through punishment are essentially irrelevant for the dynamics of reward and conversely, those effects that are essential for establishing cooperation through rewards barely affect the dynamics of punishment. Similarly, while punishment is incapable of establishing cooperation but can stabilize it, rewards can inspire cooperation but are unable to maintain it (Hilbe and Sigmund, 2010). A crucial difference between the two approaches is that once the goal is achieved, nothing needs to be done in the punishment scenario but rewarding behavior requires relentless reinforcement activities. At least in humans, positive incentives wear off over time and become ineffective—in fact, withholding rewards may be deemed a form of punishment. It remains to be seen whether the joined forces of reward, punishment and reputation indeed complement each other and manage to establish and maintain cooperation.

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