1. An ant is walking on the plane ($\mathbb{R}^2$), and experiences a temperature $f(x, y)$ given by

$$f(x, y) = \sin(x - y) + 2$$

The ant starts walking at the point $(0, 0)$.

   (a) If the ant walks east, what is the instantaneous rate of change of the temperature the ant feels? Is it getting hotter or colder?

   (b) If the ant walks north, what is the instantaneous rate of change of the temperature the ant feels? Is it getting hotter or colder?

Motivating questions:

   (a) What if the ant walks in some other direction? How can we compute the instantaneous rate of change of the temperature?

   (b) What direction should the ant walk in to experience the greatest increase in temperature?

2. Now suppose that as the ant is walking, it feels a temperature given by some unspecified function $f(x, y)$, that the ant starts at time $t = 0$ from the point (a, b) and that the ant moves with *velocity* $\mathbf{v} = \langle v_1, v_2 \rangle$.

   (a) After time t, what is the position of the ant?

   (b) What is the change in temperature the ant feels between the time 0 and the time t?

   (c) What is the average rate of change of the temperature between the time 0 and the time t?

   (d) What is the instantaneous rate of change of the temperature as the ant leaves (a, b)?

   (e) Rewrite this as the evaluation of a derivative with respect to time.

(f) Use the chain rule to express this derivative as a function of the partial derivatives of f.

(g) Rewrite this derivative as a dot product of two vectors.

Definition:

3. If $\mathbf{v} = \hat{\imath}$ (recall this is the unit vector pointing in the positive x-direction), compute $D_{\hat{\imath}}f(a, b)$. What do you notice?

4. Let $f(x, y) = e^{x^2+y}$.
   (a) What is the slope of the function at the point $P = (1, 2)$ in the direction of the vector $\langle 1, 2\rangle$?
   (b) For what direction at P will the slope be 0?
   (c) For what direction at P will the slope be maximal?

Properties of the gradient: