Definition: A set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}\}$ is **linearly independent** if the vector equation

$$x_1\mathbf{v_1} + x_2\mathbf{v_2} + \ldots + x_k\mathbf{v_k} = 0$$

has only the trivial solution.
We call the set $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}\}$ **linearly dependent** otherwise. In this case, there exist scalars $x_1, x_2, \ldots, x_k$ that are not all zero such that

$$x_1\mathbf{v_1} + x_2\mathbf{v_2} + \ldots + x_k\mathbf{v_k} = 0$$

and we call this a **linear dependence relation**.

1. Emma is out sailing again, but this time her boat is able to travel in three dimensions, along the vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$. Starting from Botany Bay, she travels for c_1 hours in the direction of the vector $\mathbf{u}$, then c_2 hours in the direction of the vector $\mathbf{v}$, then c_3 hours in the direction of the vector $\mathbf{w}$ and ends up back at Botany Bay. Is the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ linearly independent? Explain your answer.

2. For the sets in this question, determine if they are linearly independent or dependent. If they are linearly dependent, write down an explicit linear dependence relation between the vectors in the set.

   (a) $S1 = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix} \right\}$

   (b) $S2 = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ -11 \end{bmatrix} \right\}$

   (c) $S3 = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

3. Suppose $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ such that $\{\mathbf{u}, \mathbf{v}\}$ is a linearly dependent set. Is it possible for the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ to be linearly independent? Explain your answer carefully.

4. **True or False?** Let A be an $m \times n$ matrix with a pivot in every row. Then the set of columns of A is linearly independent.

5. Let $B = \begin{bmatrix} 2 & 6 & -2 & -2 & 4 & -12 \\ -1 & -3 & -3 & -1 & 0 & -2 \\ 0 & 0 & 2 & 3 & 1 & 8 \\ 1 & 3 & 4 & 2 & 0 & 5 \end{bmatrix}$.

   (a) Without doing any calculations, what is the maximum possible dimension of the span of the columns of B?

   (b) Compute the dimension of the span of the columns of B and write down a linearly independent set of vectors $S = \{\mathbf{v_1}, \ldots, \mathbf{v_k}\}$ such that $\mathrm{Span}(S)$ is equal to the span of the columns of B.