Math 100 Midterm 1 Solutions

(1)(a) $= \lim_{x\to 3} \dfrac{x(x-3)}{(x+3)(x-3)} = \lim_{x\to 3} \dfrac{x}{x+3} = \dfrac{3}{3+3} = \boxed{\dfrac{1}{2}}$

(b) $= \lim_{x\to+\infty} \dfrac{(\sqrt{x+x^2}-x)(\sqrt{x+x^2}+x)}{\sqrt{x+x^2}+x} = \lim_{x\to+\infty} \dfrac{(x+x^2)-(x)^2}{\sqrt{x+x^2}+x} = \lim_{x\to+\infty} \dfrac{x}{\sqrt{x+x^2}+x}$

$= \lim_{x\to+\infty} \dfrac{1}{\sqrt{1/x+1}+1} = \dfrac{1}{1+1} = \boxed{\dfrac{1}{2}}$

(c) $= \lim_{x\to 0} \dfrac{1}{3}\Big/\dfrac{\sin(3x)}{3x} = \dfrac{1}{3}\Big/1 = \boxed{\dfrac{1}{3}}$

(d) $= \lim_{x\to 4^+} \dfrac{4-x}{x-4} = \boxed{-1}$

because for $x > 4$ we have $x > 0$ so $|x| = x$, and $4 - x < 0$ so $|4 - x| = x - 4$.

(e) $1/(1-x^2)$ is continuous except at $x = \pm 1$, of which only $x = -1$ is relevant. e^{2x} is continuous everywhere. g continuous at 0 because $\lim_{x\to 0^-} g(x) = 1$ and $\lim_{x\to 0^+} g(x) = 1$.

$\boxed{\text{all except } x = -1}$

(f) $f'(x) = \dfrac{d}{dx}(e^x x^{1/2} + x^{-1/2}) = \boxed{e^x x^{1/2} + \tfrac{1}{2}e^x x^{-1/2} - \tfrac{1}{2}x^{-3/2}}$

(g) $\boxed{e^x - \cos x}$

(h) $g'(x) = \dfrac{2xf(x) - x^2 f'(x)}{f(x)^2}$, so $g'(1) = \dfrac{2\cdot 1\cdot 2 - 1^2\cdot 7}{2^2} = \boxed{-\dfrac{3}{4}}$

(2) The slope of the tangent at (a, a^2) is $\left.\dfrac{dy}{dx}\right|_{x=a} = 2x|_{x=a} = 2a$,

so the equation of the tangent at (a, a^2) is $y - a^2 = 2a(x - a)$, i.e.

$$y = 2ax - a^2.$$

For the tangent to pass through $(0, -4)$ we must have $-4 = 2a(0) - a^2$, so $a^2 = 4$, so $a = \pm 2$.

Substituting back into the equation, the tangents are $\boxed{y = 4x - 4 \text{ and } y = -4x - 4.}$

(3) Velocity at a time t is $h'(t) = 1 + \cos t$.

(a) $h'(t) = 0$ when $\cos t = -1$, i.e. times $\boxed{t = \pi, 3\pi, 5\pi, \ldots}$

(b) $h'(t) \geq 0$ for all t, so $\boxed{\text{never moving downwards.}}$

(c) $h(3\pi) - h(\pi) = 3\pi + \sin 3\pi - \pi - \sin\pi = \boxed{2\pi}$

(4) $f'(x) = \lim_{h\to 0} \dfrac{1/\sqrt{x+h} - 1/\sqrt{x}}{h} = \lim_{h\to 0} \dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} = \lim_{h\to 0} \dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

$= \lim_{h\to 0} \dfrac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \dfrac{-1}{\sqrt{x}\sqrt{x}(2\sqrt{x})} = -\dfrac{1}{2x^{3/2}}$

(5) Let $f(t) = 2/t + \cos(\pi t)$, and note that f is continuous except at 0.

Then $f(2) = 2/2 + 1 = 2 > 0$ and $f(3) = 2/3 - 1 = 1/3 < 0$. Since f is continuous on $[2, 3]$, by the intermediate value theorem there is a number t in $(2, 3)$ with $f(t) = 0$, that is $2/t = -\cos(\pi t)$.