

This midterm has 5 questions on 6 pages

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Full Name (Last, First): _____

Student Number: _____

Signature: _____

Question:	1	2	3	4	5	Total
Points:	10	10	9	9	12	50
Score:						

10 marks

1. (a) Write the negation of the following statement:

$$\forall x \in \mathbf{R}, \exists y \in \mathbf{R}, \text{ such that } 8x + y > 0.$$

- (b) Write the negation of the following statement:

$$\exists x \in [0, 1), \text{ such that } \sin x > 1/2 \text{ or } \sin x \leq -1/2.$$

- (c) Write the contrapositive of the following statement:

$$\text{If } x \geq -1 \text{ or } x \leq 1, \text{ then } x^2 \leq 1.$$

- (d) Let A_n be the interval $[0, 2 - \frac{1}{n}]$ for $n \in \mathbf{N}$. Find $\bigcap_{n \in \mathbf{N}} A_n$ and $\bigcup_{n \in \mathbf{N}} A_n$.

10 marks

2. Determine whether each of the following statement is True or False. You DO NOT need to justify your answer.

(a) For any set S , $\emptyset \subseteq P(S)$.

(b) For any set S , $\emptyset \in P(S)$.

(c) $\{2\} \subseteq P(\{2, 3\})$.

(d) $\{2\} \in P(\{2, 3\})$.

(e) $\{\{2\}\} \subseteq P(\{2, 3\})$.

9 marks

3. (a) Prove: If $a \equiv 0 \pmod{n}$, then for all $b \in \mathbf{Z}$, $ab \equiv 0 \pmod{n}$.

(b) Prove or disprove: Let $a, b \in \mathbf{Z}$ and $n \in \mathbf{N}$. If $ab \equiv 0 \pmod{n}$, then $a \equiv 0 \pmod{n}$ or $b \equiv 0 \pmod{n}$.

9 marks

4. Let A, B, C be sets. Let P be the statement: $A \subseteq B \implies A \cap C \subseteq B \cap C$.

(a) Is P True or False? Justify your answer.

(b) Write the converse of P . Is the converse True or False? Justify your answer.

12 marks

5. Determine whether each of the following statement is True or False. Justify your answer.

(a) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}, \text{ s.t. } xy = 1$

(b) $\exists x \in \mathbf{R}, \text{ s.t. } \forall y \in \mathbf{R} \setminus \{-1, 0\}, xy = 1$

(c) $\forall x \in \mathbf{R}, \forall y \in \mathbf{R}, \exists z \in \mathbf{R}, \text{ s.t. } x + y = z$

(d) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}, \text{ s.t. } \forall z \in \mathbf{R}, x + y = z$