- There are 10 questions worth a total of 100.

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1. 0 marks Question 1.40

Solution:

(a)

$$\begin{aligned}\bigcup_{i=1}^{5} A_{2i} &= A_2 \cup A_4 \cup A_6 \cup A_8 \cup A_{10} \\ &= \{1,3\} \cup \{3,5\} \cup \{5,7\} \cup \{7,9\} \cup \{9,11\} = \{1,3,5,\cdots,11\}.\end{aligned}$$

(b)

$$\bigcup_{i=1}^{5}(A_i \cap A_{i+1}) = \bigcup_{i=1}^{5}(\{i-1,i+1\} \cap \{i,i+2\}) = \bigcup_{i=1}^{5}\emptyset = \emptyset.$$

(c)

$$\bigcup_{i=1}^{5}(A_{2i-1} \cap A_{2i+1}) = \bigcup_{i=1}^{5}(\{2i-2,2i\} \cap \{2i,2i+2\}) = \bigcup_{i=1}^{5}\{2i\} = \{2,4,6,8,10\}.$$

2. 20 marks Question 1.42 — 10 marks for each

Solution:

(a) Let $A_n = [1, 2+1/n)$. Then

$$\bigcup_{n\in\mathbb{N}} A_n = [1,3)$$
$$\bigcap_{n\in\mathbb{N}} A_n = [1,2]$$

The union is the first interval as all the other intervals are subsets of the first interval.

The intersection contains 2 but no number bigger than 2, so it is closed. Notice that all intervals contain 2 and $2+1/n$ converges to 2 as n goes to ∞.

(b) Let $A_n = (\frac{1-2n}{n}, 2n)$. Then

$$\bigcup_{n\in\mathbb{N}} A_n = (-2,\infty)$$
$$\bigcap_{n\in\mathbb{N}} A_n = (-1,2)$$

The union contains every number larger than -2, but not -2 (since -2 does not belong to any of these intervals), so it is open.

The intersection is the first interval as the first interval is a subset of all the other intervals.

3. 15 marks Question 1.46 — 3 marks for each

Solution:

(a) S_1 is a partition of A as it satisfies all the requirements of being a partition.

(b) S_2 is not a partition of A since g belongs to no element of S_2.

(c) S_3 is a partition of A as it satisfies all the requirements of being a partition.

(d) S_4 is not partition of A since empty set belongs to S_4.

(e) S_5 is not partition of A since b belongs to two elements of S_5.

4. 10 marks Question 1.54 — 5 marks for answer, 5 marks for justification

Solution: Let $S = \{\{1\}, \{2\}, \{3, 4, 5, 6\}, \{7, 8, 9, 10\}, \{11, 12\}\}$, then S has 5 elements with no element of size 3 and it has subset $T = \{\{1\}, \{2\}, \{3, 4, 5, 6\}, \{7, 8, 9, 10\}\}$ satisfying two required conditions.

5. 0 marks Question 1.60

Solution: $\mathcal{P}(A) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, A\}$ and therefore for the cartesian product we have $A \times \mathcal{P}(A) = \{(\emptyset, \emptyset), (\emptyset, \{\emptyset\}), (\emptyset, \{\{\emptyset\}\}), (\emptyset, A), (\{\emptyset\}, \emptyset), (\{\emptyset\}, \{\emptyset\}), (\{\emptyset\}, \{\{\emptyset\}\}), (\{\emptyset\}, A)\}$.

6. 15 marks Question 2.6 — 5 marks for each

Solution: Since $A \in \mathcal{P}(\{1, 2, 4\})$, A is a subset of $\{1, 2, 4\}$.

(a) We need A to be a subset of $\{1, 2, 3\}$, so we need $A = \emptyset, \{1\}, \{2\}$ or $\{1, 2\}$.

(b) We need A to not be a subset of $\{1, 2, 3\}$, so we need $4 \in A$. Thus $A = \{4\}, \{1, 4\}, \{2, 4\}$ or $\{1, 2, 4\}$.

(c) We need A to be disjoint from $\{1, 2, 3\}$, so we need $A = \emptyset$ or $\{4\}$.

7. 0 marks Question 2.16

Solution:

(a) True.

(b) False.

(c) False.

(d) True.

(e) True.

8. 0 marks Question 2.29

Solution: Only (c) implies that $P \vee Q$ is false.

9. 20 marks Question 2.32 — 4 marks for explanation and 4 marks for each answer

Solution: To have the implication $P(x) \Rightarrow Q(x)$ true we need to look for those x which make $P(x)$ false and also those x which make both $P(x)$ and $Q(x)$ true.

(a) All $x \in S$ for which $x \neq 7$.

(b) All $x \in S$ for which $x > -1$ (this is the union of $(-1, 1)$ and $[1, +\infty)$).

(c) All $x \in S$ (for all natural numbers both hypothesis and conclusion are true).

(d) All $x \in S$ (for all elements in domain S both hypothesis and conclusion are true).

10. 20 marks Question 2.44 — 3 marks for each answer and 2 marks for each justification

Solution:

(i) $P(1) \Rightarrow Q(1)$ is false since 0 is even and 1 is odd so we have T implies F which is false;

(ii) $Q(4) \Rightarrow P(4)$ is true since 0 is even and 6 is even so we have T implies T which is true;

(iii) $P(2) \Leftrightarrow R(2)$ is true since 1 is not even and 9 is not a prime so we have F iff F which is true;

(iv) $Q(3) \Leftrightarrow R(3)$ is false since 4 is even and 33 is not a prime so we have T iff F which is false.