*This midterm has **6 questions** on **7 pages**, for a total of 100 points.*

Duration: 80 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Full Name (including all middle names): ______________________________

Student-No: ______________________________

Signature: ______________________________

Question:	1	2	3	4	5	6	Total
Points:	20	20	10	20	20	10	100
Score:							

20 marks 1. (a) (10 marks) For $A = \{-2, -1, 0, 1, 2\}$ and $B = \{-3, -2, \ldots, 2, 3\}$ list the elements of the set

$$S = \left\{ (a, b) \in A \times B \ : \ (a - b)^2 = 4 \right\}.$$

(b) (10 marks) For the sets A, B defined in part (a), give an example of a set X such that $A \cap B \subset X \subset A \cup B$.

20 marks 2. (a) (10 marks) Let P and Q be statements. Show that

$$[(P \vee Q) \wedge \sim(P \wedge Q)] \quad \equiv \sim(P \Leftrightarrow Q)\,.$$

(b) (10 marks) Prove or disprove the following statement. Let A, B, C be sets, then

$$A - (B - C) \subseteq (A - B) \cup (A \cap C)\,.$$

10 marks 3. Let $m, n \in \mathbb{Z}$. Prove that if $m \equiv n \pmod 3$ then $m^3 \equiv n^3 \pmod 9$.

20 marks 4. (a) (10 marks) Prove the following statement: for all $n \in \mathbb{Z}$, the number $n^2 + n - 1$ is odd.

(b) (10 marks) Let a, b, c be integers. Prove that if $a^2 \nmid bc$, then either $a \nmid b$ or $a \nmid c$.

20 marks 5. (a) (10 marks) Prove that every real number x satisfies $x^2 + 4 > |2x - 1|$.

(b) (10 marks) Prove the following statement: $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $y \leq x - 1 \Rightarrow y^2 - x^2 \geq 4$.

10 marks 6. Prove that the number $\sqrt{2} + 2\sqrt{3}$ is irrational. (In this question, you may use that $\sqrt{2}$ and $\sqrt{3}$ are irrational.)