MATH 226 PRACTICE MIDTERM 2

This is probably a little bit longer than the actual midterm. I will not post the solutions, but I will discuss these questions in the review class on Monday, Nov. 2.

1. The tangent plane to the surface $z = f(x, y)$ at $(x, y) = (1, 0)$ has the equation $3x - y + z = 2$. What is the directional derivative $D_{\mathbf{u}} f(1, 0)$, if $\mathbf{u} = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$?

2. A drop of water is sliding down a paraboloid $z = 25 - x^2 - 4y^2$. The gravity is causing it to slide down as rapidly as possible. Let $(x(t), y(t), z(t))$ be the path of the drop. Prove that the $x-$ and $y-$coordinates of the drop satisfy the differential equation

$$\frac{dy}{dx} = \frac{4y}{x}$$

whenever $x \neq 0$. (This equation can be solved to prove that $y = Cx^4$.)

3. Let

$$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(a) Find $\dfrac{\partial f}{\partial x}(0, 0)$ and $\dfrac{\partial f}{\partial y}(0, 0)$.

(b) Find the general formula for $D_{\mathbf{u}} f(0, 0)$, where $\mathbf{u} = (u_1, u_2)$ is a unit vector, in terms of u_1, u_2.

(c) Is f differentiable at $(0, 0)$? Why or why not?

4. Find the second order Taylor polynomial of the function $f(x, y) = \sin x \cos(2y)$ at $(\pi, \pi/2)$.

5. Find the largest and smallest values of $f(x, y) = x^2 - xy + y$ on the triangle $\{(x, y) : \ x \geq 0, \ y \geq 0, \ x + y \leq 7\}$.