*This midterm has **5 questions** on **5 pages**, for a total of 40 points.*

Duration: 50 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Full Name (Last, First, All middle names): ____________________

Student-No: ____________________

Signature: ____________________

Question:	1	2	3	4	5	Total
Points:	10	6	6	8	10	40
Score:						

10 marks 1. (a) Let

$$f(x,y) = \begin{cases} \dfrac{x^5}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Find the general formula for $D_{\mathbf{u}}f(0,0)$, where $\mathbf{u} = (u_1, u_2)$ is a unit vector, in terms of u_1, u_2.

(b) Is there a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $D_{\mathbf{u}}f(0,0) = u_1^2 - u_2^2$ for every unit vector $\mathbf{u} = (u_1, u_2)$? If yes, find it. If no, explain why.

6 marks

2. Let $w = f(a_1x + a_2y + a_3z, b_1x + b_2y + b_3z)$, where $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous first order partial derivatives. Prove that

$$c_1\frac{\partial w}{\partial x} + c_2\frac{\partial w}{\partial y} + c_3\frac{\partial w}{\partial z} = 0$$

for any vector (c_1, c_2, c_3) orthogonal to both (a_1, a_2, a_3) and (b_1, b_2, b_3).

6 marks

3. Assume that $f : \mathbb{R}^2 \to \mathbb{R}$ is a C^1 function and that the point $(1, 2, -3)$ lies on the surface $f(2x - y + z, x - z) = 10$. What condition should f satisfy so that the equation $f(2x - y + z, x - z) = 10$ could be solved for z as a differentiable function of x and y near the point $(1, 2, -3)$? (Use the Implicit Function Theorem.)

8 marks 4. Find the second order Taylor polynomial of the function $f(x, y) = \sin(x + y^2)$ at $(\pi, 0)$.

10 marks 5. Find the largest and smallest values of the function $f(x, y) = 4x - 2xy + y^2$ on the square $\{(x, y) : \ 0 \leq x \leq 2, \ 0 \leq y \leq 2\}$.