MATH 226 SAMPLE MIDTERM 1-SOLUTIONS

Fall 2014

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1. *Let* $f(x,y) = \sqrt{y - x^2}$. *Find the domain of* f, *and draw several level curves.*

   The domain is $\{(x,y) \in \mathbb{R}^2 : \; y \geq x^2\}$. The level curves $f(x,y) = c$ are parabolas $y = x^2 + c^2$.

2. *Decide whether each of the sets below is open, closed, or neither.*

   (a) $\{(x,y) \in \mathbb{R}^2 : \; 0 \leq x < 1, \; 0 \leq y < 2\}$ – neither

   (b) $\{(x,y) \in \mathbb{R}^2 : \; x + y < 2\}$ – open

3. *Find the area of the triangle in* $\mathbb{R}^3$ *with vertices* $(1,0,0)$, $(4,0,1)$, $(1,2,-1)$.

   Denote the vertices $(1,0,0)$, $(4,0,1)$, $(1,2,-1)$ by P, Q, R. Then the area is $\frac{1}{2}\|\vec{PQ} \times \vec{PR}\|$. We have:

$$\vec{PQ} \times \vec{PR} = \langle 3,0,1 \rangle \times \langle 0,2,-1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 1 \\ 0 & 2 & -1 \end{vmatrix} = -2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}.$$

   Hence the area is $\frac{1}{2}\sqrt{4+9+36} = \frac{1}{2}\sqrt{49} = \frac{7}{2}$.

4. *Find the scalar parametric equations of the line which passes through the point* $(4,5,-2)$ *and is perpendicular to the plane through the three points* $(1,0,1), (3,2,0), (-1,1,2)$.

   The line should be parallel to the normal vector to the plane in question. Denote the three points in the plane by P, Q, R. Then the normal vector is

$$\mathbf{n} = \vec{PQ} \times \vec{PR} = \langle 2,1,-1 \rangle \times \langle -2,1,1 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ -2 & 1 & 1 \end{vmatrix} = 3\mathbf{i} + 6\mathbf{k}.$$

   Hence the vector parametric equation is $\mathbf{r} = (4+3t)\mathbf{i} + 5\mathbf{j} + (6t-2)\mathbf{k}$, and the scalar parametric equations are

$$x = 4 + 3t, \quad y = 5, z = 6t - 2.$$

5. *Let*

$$f(x,y) = \begin{cases} \dfrac{x^3 + xy - y^3}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   (a) Find $\dfrac{\partial f}{\partial x}(0,0)$ *and* $\dfrac{\partial f}{\partial y}(0,0)$.

   We have $f(x,0) = \dfrac{x^3}{x^2} = x$ and $f(0,y) = \dfrac{-y^3}{y^2} = -y$, hence $\dfrac{\partial f}{\partial x}(0,0) = 1$ and $\dfrac{\partial f}{\partial y}(0,0) = -1$.

   (b) Does f *have a limit at* $(0,0)$*? Explain your answer.*

   From part (a), $\lim_{x\to 0} f(x,0) = \lim_{y \to 0} f(0,y) = 0$. But on the other hand, $\lim_{x \to 0} f(x,x) = \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$. Since f approaches different values as (x,y) approaches $(0,0)$ along different trajectories, the limit does not exist.

6. *Find all values of* (a,b) *such that the tangent plane to the surface* $z = 4x^2 - y^2$ *at* $(a, b, 4a^2 - b^2)$ *is parallel to the line with parametric equations* $x = t+1$, $y = 2t$, $z = -4t+9$.

Let $f(x,y) = 4x^2 - y^2$, then $f_x(a,b) = 8a$ and $f_y(a,b) = -2b$. Hence the tangent plane at $(a, b, 4a^2 - b^2)$ is perpendicular to the vector $\mathbf{n} = -8a\mathbf{i} + 2b\mathbf{j} + \mathbf{k}$. We want $\mathbf{n}$ to be perpendicular to the direction vector $\mathbf{v}$ of the given line. From the equations of the line, $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. We therefore want $\mathbf{n} \cdot \mathbf{v} = 0$, so that $-8a + 4b - 4 = 0$, or simplifying, $-2a + b = 1$. All values (a,b) with $b = 1 + 2a$ satisfy this condition.