*This midterm has **5 questions** on **6 pages**, for a total of 50 points.*

Duration: 50 minutes

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Last name: ______________________ Student No.: ______________

First name and all middle names: ______________________

Signature: ______________________

Question:	1	2	3	4	5	Total
Points:	12	8	8	12	10	50
Score:						

12 marks 1. Let

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (8 marks) Does f have a limit at $(0,0)$? Explain your answer.

(b) (4 marks) Find $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$, or else explain why they do not exist.

8 marks 2. Find the equation of the tangent plane to the surface $z = 5x^2y - y^2$ at the point where $x = -1$, $y = 3$.

8 marks 3. Let $w = f(a_1x + a_2y + a_3z, b_1x + b_2y + b_3z)$, where $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous first order partial derivatives. Prove that

$$c_1\frac{\partial w}{\partial x} + c_2\frac{\partial w}{\partial y} + c_3\frac{\partial w}{\partial z} = 0$$

for any vector (c_1, c_2, c_3) orthogonal to both (a_1, a_2, a_3) and (b_1, b_2, b_3).

12 marks 4. The tangent plane to the graph of $z = f(x, y)$ at $(x, y) = (3, 4)$ has the equation $x - 6y - 2z = 1$.

(a) Find the directional derivative $D_{\mathbf{u}}f(3, 4)$ if $\mathbf{u} = \frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$.

(b) Is there a unit vector $\mathbf{v}$ such that $D_{\mathbf{v}}f(3, 4) = 4$? If yes, find it. If no, explain why.

10 marks 5. The equations

$$u = x^3 - y^2$$
$$v = 2xy^3$$

define x, y implicitly as functions of u, v near $x = -1$, $y = 1$. Find $\dfrac{\partial x}{\partial u}$ and $\dfrac{\partial y}{\partial u}$ at $x = -1$, $y = 1$.

This page has been left blank for your workings and solutions.