$\vec{F} = (F_1, F_2, F_3)$

We have a stream of particles.

β = number of particles per unit volume
= particle density

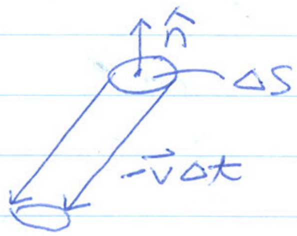
each particle is m kg. ← mass

$\Rightarrow$ mass density $\mu = m\beta$ $\frac{\text{kg}}{\text{unit VOL}}$.

$\vec{v}(x,y,z)$ is velocity of particle located at (x,y,z)
($\vec{v}$ is the velocity field)

$\vec{F}(x,y,z) = \mu(x,y,z)\,\vec{v}(x,y,z)$ is
mass flow rate density of stream

① If we count particles crossing ΔS for next Δt seconds, which will be the Last particles to go through?
They are the ones
$-(\vec{v}\,\Delta t)$ away from the Patch



$\hat{n}$ outward unit normal

② How many are inside cylinder?
VOL = (base area) × height
$= (\Delta S)\ \hat{n}\cdot\vec{v}\,\Delta t$
$\Rightarrow$ it contains
$\beta\,(\Delta S)\ \hat{n}\cdot\vec{v}\,\Delta t$ particles

③ The mass flow rate through the small area ΔS per unit time
is $(m\beta\vec{v})\cdot\hat{n}(\Delta S) = \mu\vec{v}\cdot\hat{n}\,\Delta S$
$= \vec{F}\cdot\hat{n}\,\Delta S$

This is the flux of the vector field $\vec{F}$ through ΔS.