Practice Midterm **October 15, 2015** **Duration: 50 minutes**

*This test has **5 questions** on **8 pages**, for a total of 40 points.*

- Read all the questions carefully before starting to work.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **No aids of any kind are allowed**, including: documents, cheat sheets, electronic devices (including calculators, phones, etc.)

First Name: ______________________ Last Name: ______________________

Student No.: ______________________ Section: ______________________

Signature: ____

Question:	1	2	3	4	5	Total
Points:	10	10	4	8	8	40
Score:						

1. Let $\{s_n\}$ be a sequence in $\mathbb{C}$.

2 marks (a) Define what it means to say that $\lim_{n\to\infty} s_n = s$.

3 marks (b) Suppose $\lim_{n\to\infty} s_n = s$. Prove that the sequence $\{s_n\}$ is bounded.

5 marks

(c) Let $\sigma_n = \frac{1}{n}(s_1 + \cdots + s_n)$. Suppose $\lim_{n\to\infty} s_n = s$. Prove that $\lim_{n\to\infty} \sigma_n = s$.

2 marks

2. (a) Suppose $E \subset \mathbb{R}$ is nonempty and bounded above. Define precisely what it means to say that $x = \sup E$.

8 marks

(b) Let A and B be subsets of $[0, \infty)$ which are bounded above, i.e., bounded subsets of non-negative real numbers. Let $AB = \{xy : x \in A, y \in B\}$. Prove that $\sup(AB) = (\sup A)(\sup B)$.

4 marks

3. Suppose that A, B are subsets of a metric space X. We say that A is dense in B iff B is contained in the closure of A. Let $C \subset X$, and suppose that A is dense in B, and that B is dense in C. Prove that A is dense in C.

8 marks 4. True or False. If True, give a proof; if False, provide a counter-example.

(a) The set of infinite subsets of a countable set is uncountable.

(b) A surjective map from the natural numbers to a countable set must also be injective.

8 marks

5. Let $\{a_n\}$ be a sequence of real numbers, and let $a \in \mathbb{R}$. Suppose that every subsequence of $\{a_n\}$ has a subsequence convergent to a. Prove that $\{a_n\}$ converges to a.

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