

Practice Midterm October 15, 2015 Duration: 50 minutes*This test has 5 questions on 8 pages, for a total of 40 points.*

- Read all the questions carefully before starting to work.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **No aids of any kind are allowed**, including: documents, cheat sheets, electronic devices (including calculators, phones, etc.)

First Name: _____ Last Name: _____

Student No.: _____ Section: _____

Signature: _____

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|-----------|----|----|---|---|---|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points: | 10 | 10 | 4 | 8 | 8 | 40 |
| Score: | | | | | | |

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (i) speaking or communicating with other examination candidates, unless otherwise authorized;
 - (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
 - (iii) purposely viewing the written papers of other examination candidates;
 - (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)?(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1. Let $\{s_n\}$ be a sequence in \mathbb{C} .

2 marks

(a) Define what it means to say that $\lim_{n \rightarrow \infty} s_n = s$.

3 marks

(b) Suppose $\lim_{n \rightarrow \infty} s_n = s$. Prove that the sequence $\{s_n\}$ is bounded.

5 marks

(c) Let $\sigma_n = \frac{1}{n}(s_1 + \cdots + s_n)$. Suppose $\lim_{n \rightarrow \infty} s_n = s$. Prove that $\lim_{n \rightarrow \infty} \sigma_n = s$.

2 marks

2. (a) Suppose $E \subset \mathbb{R}$ is nonempty and bounded above. Define precisely what it means to say that $x = \sup E$.

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|---------|
| 8 marks |
|---------|

- (b) Let A and B be subsets of $[0, \infty)$ which are bounded above, i.e., bounded subsets of non-negative real numbers. Let $AB = \{xy : x \in A, y \in B\}$. Prove that $\sup(AB) = (\sup A)(\sup B)$.

| |
|---------|
| 4 marks |
|---------|

3. Suppose that A, B are subsets of a metric space X . We say that A is dense in B iff B is contained in the closure of A . Let $C \subset X$, and suppose that A is dense in B , and that B is dense in C . Prove that A is dense in C .

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|---------|
| 8 marks |
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4. True or False. If True, give a proof; if False, provide a counter-example.

(a) The set of infinite subsets of a countable set is uncountable.

(b) A surjective map from the natural numbers to a countable set must also be injective.

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| 8 marks |
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5. Let $\{a_n\}$ be a sequence of real numbers, and let $a \in \mathbb{R}$. Suppose that every subsequence of $\{a_n\}$ has a subsequence convergent to a . Prove that $\{a_n\}$ converges to a .

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