**MATHEMATICS 542, PROBLEM SET 1**
**Due on Friday, 17 October 2008**

1. Prove that the number of $k$-tuples of integers $a_1, \dots, a_k$ such that $1 \le a_1 \le a_2 \le \cdots \le a_k \le n$ is equal to $\binom{n+k-1}{k}$.

2. Let $A \subset \mathbb{Z}_N$, $|A| \le \frac{1}{10} \log N$. Prove that there is a $\xi \ne 0$ such that $|\widehat{A}(\xi)| \ge |A|/(2N)$.

3. Prove that for every $C > 0$ there is a $N = N(C)$ such that if $A \subset \mathbb{Z}$, $|A| = N$ and $|A + A| \le CN$, then $A$ contains a non-trivial 3-term arithmetic progression.

4. Let $A \subset \mathbb{Z}_N$, $|A| = \delta N$. Prove that $3A$ contains a (mod $N$) arithmetic progression of length $N^c$ for some $c$ depending only on $\delta$.

5. Let $N$ be prime, $A \subset \mathbb{Z}_N$, $|A| > N/2$. Suppose that $\phi : A \to B$, $B \subset \mathbb{Z}_N$, is a Freiman 2-homomorphism. Must $\phi$ have the form $\phi(x) = ax + b$ for some $a, b \in \mathbb{Z}_N$? If yes, prove it; if no, give an example.

6. Modify Behrend's example to prove the following: it is possible to colour the integers $\{1, 2, \dots, N\}$ using no more than $\exp(c\sqrt{\log N})$ colours (with $c$ independent of $N$) so that there are no monochromatic 3-term arithmetic progressions.