Lab 3: Acceleration, velocity, displacement

1 Introduction

The purpose of this lab is to show the connection between acceleration, a, velocity, v, and displacement, x. You should read the course notes to familiarize yourself with the relationships (in terms of anti-differentiation). Here we will use the fact that

$$v \approx \sum a(t)\Delta t \quad \text{and} \quad x \approx \sum v(t)\Delta t$$

to calculate (using the spreadsheet) an approximation for the velocity and the displacement, given the acceleration of some particle. Although the calculation can also be done by integration, our point here is to show the numerical version that uses the spreadsheet, so we have chosen acceleration functions $a(x)$ which are difficult to integrate by hand.

2 Problems

2.1 Problem Set 1

A particle accelerates from rest, its acceleration depending on time as follows:

$$a(t) = \frac{t^{2.7}}{4+t^3}\,\frac{m}{s^2}.$$

Use the spreadsheet to compute the velocity of the particle over the time interval $0 \le t \le 4$. Determine the displacement of the particle over the same time interval. How far did the particle move altogether, and what were its velocity and its acceleration at $t = 4$?
You should hand in one page showing

- The graphs of acceleration, velocity, and position over the time interval $0 \le t \le 4$.
- The values you find for the final acceleration, velocity, and displacement (at time $t = 4$), accurate to 1 decimal place.

We suggest that you use the approximation techniques of Lab 2 to compute the necessary integrals. The accuracy of your approximation is controlled by the step size Δt. If you are worried that your results are not accurate enough, then decrease your step size and see how much your final values change.

You may find it helpful to use the "copy" command, under the "edit" menu, to make long columns of numbers in Mathsheet.

2.1.1 Solution to Problem Set 1

We put time values in column a, using a relatively small step size of $\Delta t = 0.01$ for good accuracy. The given acceleration as a function of time is recorded in column b. We calculate the velocity and displacement in columns c and d in the spreadsheet using the approximations

$$v(t) \approx \sum a(t')\Delta t',$$

$$x(t) \approx \sum v(t')\Delta t'.$$

The results are shown in the figure below. According to the spreadsheet calculation at time $t = 4$ we have

$$a(4) = 0.621, \qquad v(4) = 1.674, \qquad x(4) = 2.349.$$

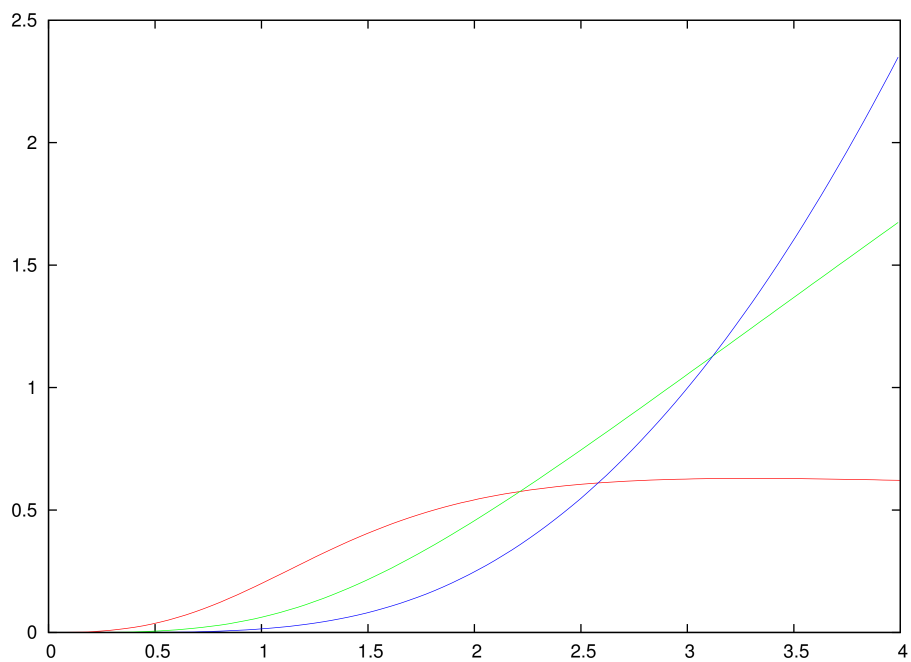

Figure 1 The given acceleration, together with the approximate velocity and displacement are shown here for results of lab 3 problem set 1. Acceleration is plotted in red, velocity in green, and position in blue.

2.2 Problem Set 2

A particle accelerates from rest, its acceleration depending on time as follows:

$$a(t) = \frac{t^{3.2}}{2+t^2}\ \frac{m}{s^2}.$$

Use the spreadsheet to compute the velocity of the particle over the time interval $0 \leq t \leq 3$. Determine the displacement of the particle over the same time interval. How far did the particle move altogether, and what were its velocity and its acceleration at $t = 3$?
You should hand in one page showing

- The graphs of acceleration, velocity, and position over the time interval $0 \leq t \leq 3$.
- The values you find for the final acceleration, velocity, and displacement (at time $t = 3$), accurate to 1 decimal place.

We suggest that you use the approximation techniques of Lab 2 to compute the necessary integrals. The accuracy of your approximation is controlled by the step size Δt. If you are worried that your results are not accurate enough, then decrease your step size and see how much your final values change.

You may find it helpful to use the "copy" command, under the "edit" menu, to make long columns of numbers in Mathsheet.

2.2.1 Solution to Problem Set 2

We put time values in column a, using a relatively small step size of $\Delta t = 0.01$ for good accuracy. The given acceleration as a function of time is recorded in column b. We calculate the velocity and displacement in columns c and d in the spreadsheet using the approximations

$$v(t) \approx \sum a(t')\Delta t',$$

$$x(t) \approx \sum v(t')\Delta t'.$$

The results are shown in the figure below. According to the spreadsheet calculation at time $t = 3$ we have

$$a(3) = 3.042, \qquad v(3) = 3.24, \qquad x(3) = 2.437.$$

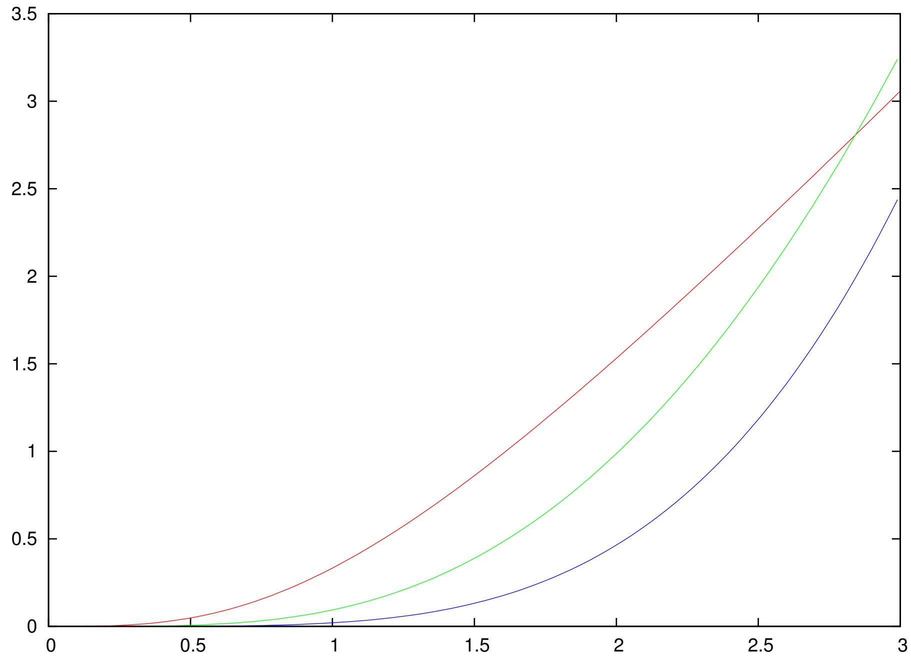

Figure 2 The given acceleration, together with the approximate velocity and displacement are shown here for results of lab 3 problem set 2. Acceleration is plotted in red, velocity in green, and position in blue.

2.3 Problem Set 3

A particle accelerates from rest, its acceleration depending on time as follows:

$$a(t) = \frac{3t^{2.5}}{1+3t^3}\ \frac{m}{s^2}.$$

Use the spreadsheet to compute the velocity of the particle over the time interval $0 \le t \le 3$. Determine the displacement of the particle over the same time interval. How far did the particle move altogether, and what were its velocity and its acceleration at $t = 3$?
You should hand in one page showing

- The graphs of acceleration, velocity, and position over the time interval $0 \le t \le 3$.
- The values you find for the final acceleration, velocity, and displacement (at time $t = 3$), accurate to 1 decimal place.

We suggest that you use the approximation techniques of Lab 2 to compute the necessary integrals. The accuracy of your approximation is controlled by the step size Δt. If you are worried that your results are not accurate enough, then decrease your step size and see how much your final values change.

You may find it helpful to use the “copy” command, under the “edit” menu, to make long columns of numbers in Mathsheet.

2.3.1 Solution to Problem Set 3

We put time values in column a, using a relatively small step size of $\Delta t = 0.01$ for good accuracy. The given acceleration as a function of time is recorded in column b. We calculate the velocity and displacement in columns c and d in the spreadsheet using the approximations

$$v(t) \approx \sum a(t')\Delta t',$$

$$x(t) \approx \sum v(t')\Delta t'.$$

The results are shown in the figure below. According to the spreadsheet calculation at time $t = 3$ we have

$$a(3) = 0.571, \qquad v(3) = 1.726, \qquad x(3) = 2.284.$$

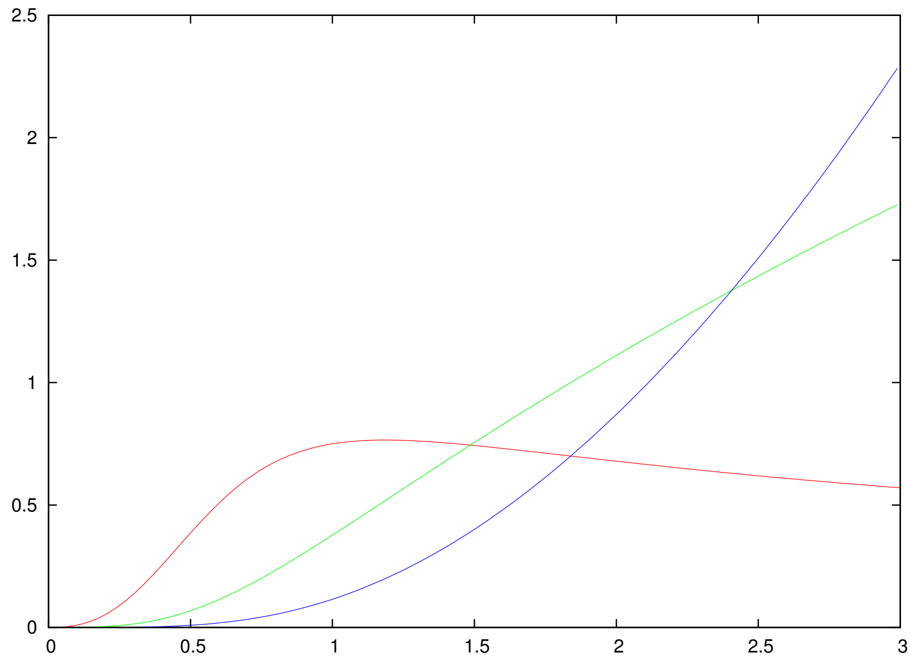

Figure 3 The given acceleration, together with the approximate velocity and displacement are shown here for results of lab 3 problem set 3. Acceleration is plotted in red, velocity in green, and position in blue.

2.4 Problem Set 4

A particle accelerates from rest, its acceleration depending on time as follows:

$$a(t) = \frac{2t^{2.8}}{3 + 2t^3}\ \frac{m}{s^2}.$$

Use the spreadsheet to compute the velocity of the particle over the time interval $0 \le t \le 3.5$. Determine the displacement of the particle over the same time interval. How far did the particle move altogether, and what were its velocity and its acceleration at $t = 3.5$?
You should hand in one page showing

- The graphs of acceleration, velocity, and position over the time interval $0 \le t \le 3.5$.
- The values you find for the final acceleration, velocity, and displacement (at time $t = 3.5$), accurate to 1 decimal place.

We suggest that you use the approximation techniques of Lab 2 to compute the necessary integrals. The accuracy of your approximation is controlled by the step size Δt. If you are worried that your results are not accurate enough, then decrease your step size and see how much your final values change.

You may find it helpful to use the "copy" command, under the "edit" menu, to make long columns of numbers in Mathsheet.

2.4.1 Solution to Problem Set 4

We put time values in column a, using a relatively small step size of $\Delta t = 0.01$ for good accuracy. The given acceleration as a function of time is recorded in column b. We calculate the velocity and displacement in columns c and d in the spreadsheet using the approximations

$$v(t) \approx \sum a(t')\Delta t',$$

$$x(t) \approx \sum v(t')\Delta t'.$$

The results are shown in the figure below. According to the spreadsheet calculation at time $t = 3.5$ we have

$$a(3.5) = 0.752, \qquad v(3.5) = 1.874, \qquad x(3.5) = 2.405.$$

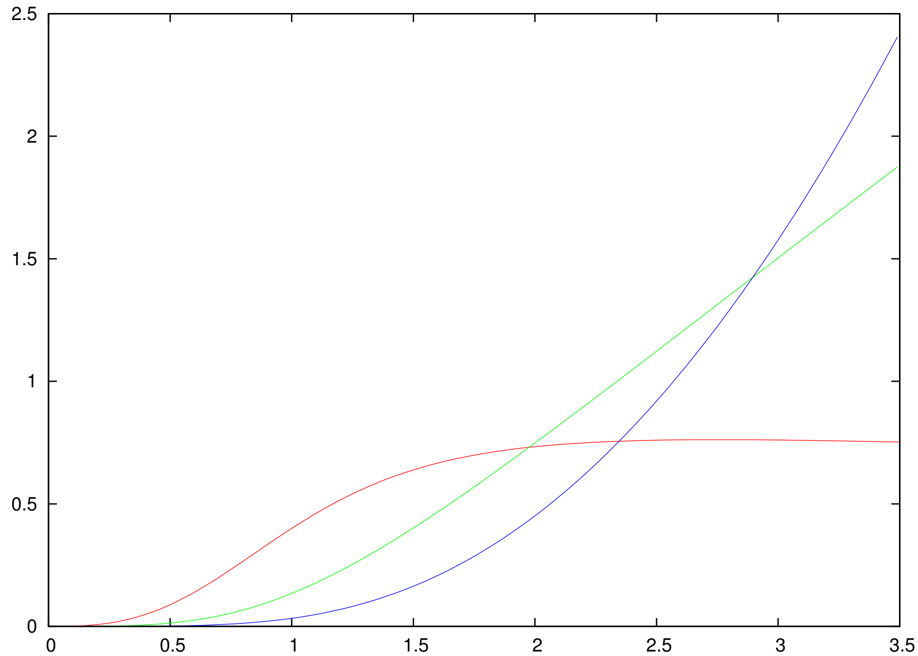

Figure 4 The given acceleration, together with the approximate velocity and displacement are shown here for results of lab 3 problem set 4. Acceleration is plotted in red, velocity in green, and position in blue.

2.5 Problem Set 5

A particle accelerates from rest, its acceleration depending on time as follows:

$$a(t) = \frac{2t^{3.1}}{4+3t^2}\ \frac{m}{s^2}.$$

Use the spreadsheet to compute the velocity of the particle over the time interval $0 \leq t \leq 3.5$. Determine the displacement of the particle over the same time interval. How far did the particle move altogether, and what were its velocity and its acceleration at $t = 3.5$?
You should hand in one page showing

- The graphs of acceleration, velocity, and position over the time interval $0 \leq t \leq 3.5$.
- The values you find for the final acceleration, velocity, and displacement (at time $t = 3.5$), accurate to 1 decimal place.

We suggest that you use the approximation techniques of Lab 2 to compute the necessary integrals. The accuracy of your approximation is controlled by the step size Δt. If you are worried that your results are not accurate enough, then decrease your step size and see how much your final values change.

You may find it helpful to use the "copy" command, under the "edit" menu, to make long columns of numbers in Mathsheet.

2.5.1 Solution to Problem Set 5

We put time values in column a, using a relatively small step size of $\Delta t = 0.01$ for good accuracy. The given acceleration as a function of time is recorded in column b. We calculate the velocity and displacement in columns c and d in the spreadsheet using the approximations

$$v(t) \approx \sum a(t')\Delta t',$$

$$x(t) \approx \sum v(t')\Delta t'.$$

The results are shown in the figure below. According to the spreadsheet calculation at time $t = 3.5$ we have

$$a(3.5) = 2.376, \qquad v(3.5) = 3.321, \qquad x(3.5) = 3.176.$$

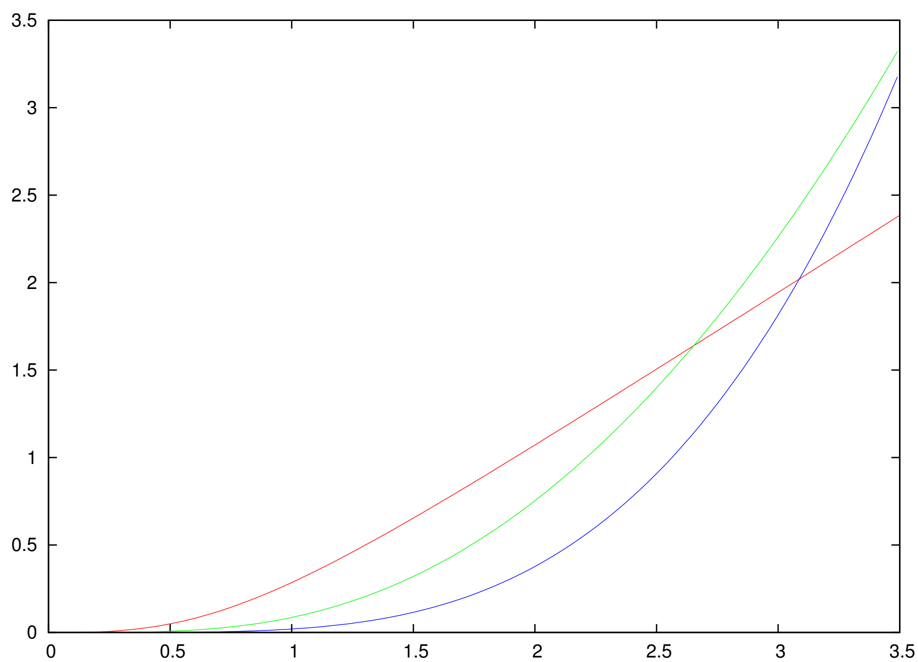

Figure 5 The given acceleration, together with the approximate velocity and displacement are shown here for results of lab 3 problem set 5. Acceleration is plotted in red, velocity in green, and position in blue.