Cohomology of moduli spaces of curves

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Intercontinental Moduli and Algebraic Geometry Zoominar
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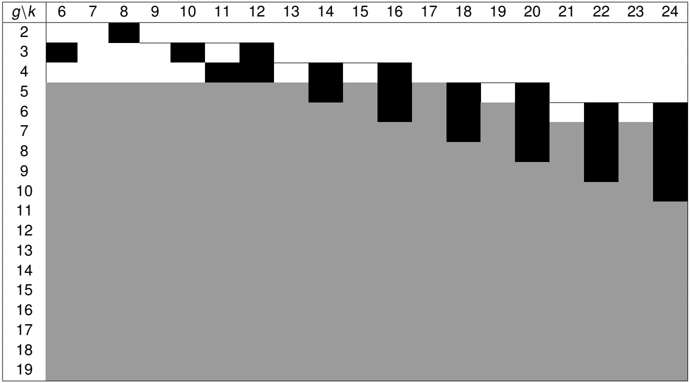
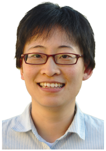
The first two unstable classes

1993, 2005

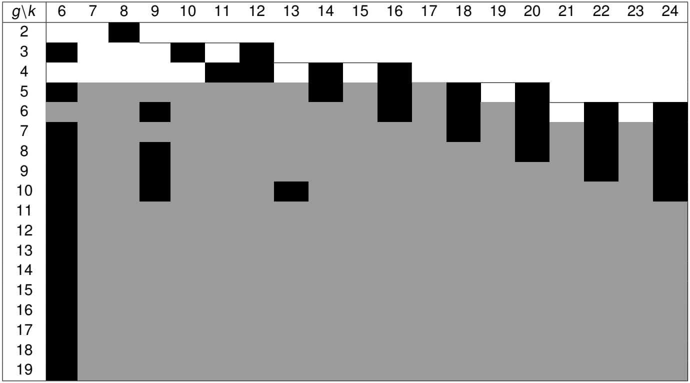


$$H^{4g-k}(\mathcal{M}_g)$$

An infinite family of unstable classes



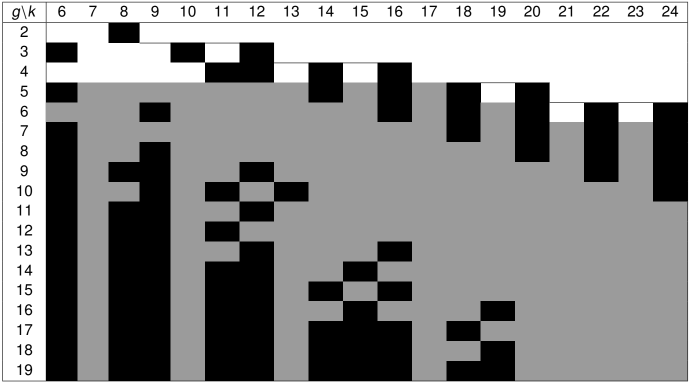
2021



$$H^{4g-k}(\mathcal{M}_g)$$

More infinite families

2022



$$H^{4g-k}(\mathcal{M}_g)$$

An open problem

Theorem (P-Willwacher 2023)

$\dim_{\mathbb{Q}} H^{4g-k}(\mathcal{M}_g)$ grows at least exponentially with g for each $6 \leq k \leq 59$ except possibly for $k \in \{7, 10, 13, 26, 57\}$.

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Conjecture

The dimension of $H^{4g-k}(\mathcal{M}_g)$ grows at least exponentially with g for all but finitely many non-negative integers k.

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Question

Does $H^{4g-7}(\mathcal{M}_g) = 0$ for all g?