**MATH256-103 Homework Assignment 1 (Due Date: September 17, 2018)**

Hand in your homework either in class, or by 6pm at my office, on September 17. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. In the falling objects example, the equation for the velocity is

$$\frac{dv}{dt} = 9.8 - 2v, v(0) = 0$$

a) Find the velocity. b) Let $x$ be the displacement from the starting point. Use the equation $v = \frac{dx}{dt}, x(0) = 0$ to find $x = x(t)$. c) Suppose the object is placed $100m$. Find the time $T$ when it hits the ground.

2. Consider the following ODE

$$y^{''} + 3y^{'} - 4y = 0$$

(a) Using $y = e^{rt}$ to find two solutions $y_1, y_2$. (b) Find the constants $a$ and $b$ such that $y = ay_1 + by_2$ solves

$$y^{''} + 3y^{'} - 4y = 0, y(0) = 1, y^{'}(0) = -1$$

2. Find the solutions to

a) $\frac{dy}{dt} = 2y - 5, y(0) = y_0$; b) $\frac{dy}{dt} + 8y = 10, y(0) = y_0$

In both cases determine how solutions behave as $t \to +\infty$.

3. Fine the general solutions to the following first order linear equations

a) $y^{'} + y = e^{-t}, y(0) = 1$; b) $ty^{'} + y = 3t\cos(2t)$; c) $2y^{'} + y = 3t^2$; (d) $ty^{'} + (t+1)y = t$

4. Find the solutions to the following initial value problem and state the interval of existence

a) $y^{'} - y = 2te^t, y(0) = 1$; b) $y^{'} + \frac{2}{t}y = \frac{\cos t}{t^2}, y(\pi) = 0, t > 0$; c) $y^{'} - y = t - \sin t + e^{2t}, y(0) = 0$ d) $\sin t y^{'} - 2\cos t y = \sin^3 t, y(\frac{\pi}{2}) = 1$

5. Consider the initial value problem

$$y^{'} + 2y = 3 + 2\cos(2t), y(0) = 0$$

Find the solution and describe the asymptotic behavior for large $t$.

6. (a) Consider the following first order ODE

$$y^{'} + y = e^{-2t}$$

Show that all solutions have the property that $y(t) \to 0$ as $t \to +\infty$.

(b) Now consider the following first order ODE

$$y^{'} - y = e^{-2t}$$

Show that there exists only one solution with the property that $y(t) \to 0$ as $t \to +\infty$. Find $y(0) = y_0$ for that solution.

7. Solve the following ODE

a) $y^{'} + y^2 \sin x = 0$; b) $y^{'} = \frac{x - e^{-x}}{y + e^y}$; c) $y^{'} = \frac{1+y^2}{2+x^2}$; d) $y^{'} = \frac{x}{y}$

8. Solve the following ODE and state the interval of existence

a) $y^{'} = (1 - 2x)y^2, y(0) = 1$; (b) $y^{'} = xy^3(1 + x^2)^{-1/2}, y(0) = 2$; c)$y^{'} = (3x^2 - e^x)/(2y - 5), y(0) = 1$; d) $\sin(2x)dx + \cos(3y)dy = 0, y(\frac{\pi}{2}) = \pi/3$.