Be sure this exam has 8 pages including the cover

The University of British Columbia

MATH 256, Section 201

Midterm Exam I – Feb. 8 2018

Name ______________________ **Signature** ______________________

Student Number ____________________

This exam consists of **6** questions. No notes. Simple numerics calculators are allowed. Write your answer in the blank page provided.

Problem	max score	score
1.	15	
2.	20	
3.	15	
4.	15	
5.	20	
6.	15	
total	100	

(15 points) 1. Consider the following ordinary differential equation

$$ty' + 2y = t^{-1}e^{t}, \qquad y(1) = 1$$

(2 points) (a) Write the equation in the following form

$$y' + p(t)y = g(t).$$

(6 points) (b) Compute $\mu(t) = e^{\int p(t)dt}$ and $\int \mu(t)g(t)dt$.

(7 points) (c) Find the solution and state the Interval of Existence.

(20 points) 2. Solve the following ordinary differential equation

$$y' = \frac{t}{2(y - y^3)}, \quad y(0) = -2$$

and state the Interval of Existence.

(15 points) 3. Consider the following ordinary differential equation

$$y' = (y-4)\log y, \;\; y > 0.$$

(10 points) (a) Find all critical points and classify the stability/instability of these critical points.

(3 points) (b) Let $y(0) = \frac{1}{2}$. What is the asymptotic behavior of $y(t)$ as $t \to +\infty$?

(2 points) (c) Let $y(0) = 2$. What is the asymptotic behavior of $y(t)$ as $t \to +\infty$?

(15 points) 4. Consider the following ordinary differential equation

$$t^2y'' - ty' + y = 0$$

(2 points) (a) Write the equation in the following form:

$$y'' + p(t)y' + q(t)y = 0$$

(5 points) (b) Find the Wronskian W.

(8 points) (c) Let $y_1 = t$ be a solution. Use reduction of order to find $y_2(t) = v(t)y_1(t)$. Hint: you may use the formula: $v' = \frac{W}{y_1^2}$.

(20 points) 5. Consider the following second order ordinary differential equation:

$$y'' - y' - 2y = h(t)$$

(5 points) (a) Find the solutions to the homogeneous problem

$$y'' - y' - 2y = 0.$$

(5 points) (b) Suppose $h(t) = \cos(t) + 2e^t$. Use the method of undetermined coefficients to find the form of the special solution y_p. Do not attempt to find the coefficients.

(5 points) (c) Suppose $h(t) = te^{2t}$. Use the method of undetermined coefficients to find the form of the special solution y_p. Do not attempt to find the coefficients.

(5 points) (d) Solve the following second order differential equation

$$y'' - y' - 2y = t, y(0) = 0, y'(0) = 1$$

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(15 points) 6. Use the method of variation of parameters to solve the inhomogeneous problem

$$y'' + 9y = \frac{3}{\cos(3t)}, -\frac{\pi}{6} < t < \frac{\pi}{6}.$$

Hint: You may use the formula $\int \frac{\sin u}{\cos u} du = -\log \cos u + C$.