

Be sure this exam has 8 pages including the cover

The University of British Columbia

MATH 256, Section 201

Midterm Exam I – Feb. 8 2018

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_

This exam consists of 6 questions. No notes. Simple numerics calculators are allowed. Write your answer in the blank page provided.

| Problem | max score | score |
|---------|-----------|-------|
| 1.      | 15        |       |
| 2.      | 20        |       |
| 3.      | 15        |       |
| 4.      | 15        |       |
| 5.      | 20        |       |
| 6.      | 15        |       |
| total   | 100       |       |

**1. Each candidate should be prepared to produce his library/AMS card upon request.**

**2. Read and observe the following rules:**

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

**3. Smoking is not permitted during examinations.**

(15 points) 1. Consider the following ordinary differential equation

$$ty' + 2y = t^{-1}e^t, \quad y(1) = 1$$

(2 points) (a) Write the equation in the following form

$$y' + p(t)y = g(t).$$

(6 points) (b) Compute  $\mu(t) = e^{\int p(t)dt}$  and  $\int \mu(t)g(t)dt$ .

(7 points) (c) Find the solution and state the Interval of Existence.

(20 points) 2. Solve the following ordinary differential equation

$$y' = \frac{t}{2(y - y^3)}, \quad y(0) = -2$$

and state the Interval of Existence.

(15 points) 3. Consider the following ordinary differential equation

$$y' = (y - 4) \log y, \quad y > 0.$$

(10 points) (a) Find all critical points and classify the stability/instability of these critical points.

(3 points) (b) Let  $y(0) = \frac{1}{2}$ . What is the asymptotic behavior of  $y(t)$  as  $t \rightarrow +\infty$ ?

(2 points) (c) Let  $y(0) = 2$ . What is the asymptotic behavior of  $y(t)$  as  $t \rightarrow +\infty$ ?

(15 points) 4. Consider the following ordinary differential equation

$$t^2 y'' - t y' + y = 0$$

(2 points) (a) Write the equation in the following form:

$$y'' + p(t)y' + q(t)y = 0$$

(5 points) (b) Find the Wronskian  $W$ .

(8 points) (c) Let  $y_1 = t$  be a solution. Use reduction of order to find  $y_2(t) = v(t)y_1(t)$ . Hint: you may use the formula:  $v' = \frac{W}{y_1^2}$ .

(20 points) 5. Consider the following second order ordinary differential equation:

$$y'' - y' - 2y = h(t)$$

(5 points) (a) Find the solutions to the homogeneous problem

$$y'' - y' - 2y = 0.$$

(5 points) (b) Suppose  $h(t) = \cos(t) + 2e^t$ . Use the method of undetermined coefficients to find the form of the special solution  $y_p$ . Do not attempt to find the coefficients.

(5 points) (c) Suppose  $h(t) = te^{2t}$ . Use the method of undetermined coefficients to find the form of the special solution  $y_p$ . Do not attempt to find the coefficients.

(5 points) (d) Solve the following second order differential equation

$$y'' - y' - 2y = t, y(0) = 0, y'(0) = 1$$

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(15 points) 6. Use the method of variation of parameters to solve the inhomogeneous problem

$$y'' + 9y = \frac{3}{\cos(3t)}, -\frac{\pi}{6} < t < \frac{\pi}{6}.$$

Hint: You may use the formula  $\int \frac{\sin u}{\cos u} du = -\log \cos u + C$ .