MATH305-201-2021/2022 Homework Assignment 1 (Due Date: Jan. 17, 2022)

10pts each

1. Calculate the following complex numbers in the form $a + bi$:
   (a) $(1+i)(3-2i)(2+3i)$; (b) $(\frac{1+i}{2+i})^2$; (c) $(1+i)^8$

2. Prove that if $|z| = 1 (z \neq 1)$, then $Re(\frac{1}{1-z}) = \frac{1}{2}$. Here $Re(w)$ denotes the real part of w.

3. Find the followings (for 3(d) write your answer in terms of $arctan$):
   (a) $|\frac{(\sqrt{3}+i)^{100}}{(\sqrt{3}-i)^{100}}|$; (b) $Arg(-1-\sqrt{3}i)$; (c) $arg(1-\sqrt{3}i)$; (d) $arg(-1+2i)$

4. Find the principal argument of each of the following complex numbers and write each in polar form.
   (a) $-3+3i$; (b) $\frac{1-i}{-\sqrt{3}+i}$; (c) $(\sqrt{3}-i)^2$

5. Decide which of the following statements are always true.
   (a) $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$ if $z_1 \neq 0, z_2 \neq 0$
   (b) $Arg(\bar{z}) = -Arg(z)$ if z is not a real number.
   (c) $arg(\bar{z}) = -arg(z)$.
   (c) $arg(z) = Arg(z) \pm 2\pi k, k = 0, 1, 2, ...$ if $z \neq 0$

6. Use De Moivre's formula together with binomial formula and geometric sequence formula to prove
   (a) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$
   (b) $1 + \cos\theta + ... + \cos n\theta = \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin(\frac{\theta}{2})}$

7. Use De Moivre's formula and binomial formula to compute
   (a) $\int_0^{2\pi} \cos^6\theta d\theta$; (b) $\int_0^{2\pi} \sin^6(2\theta) d\theta$

8. Describe the set of points z in the complex plane that satisfies each of the following
   (a) $|z-1-i| = |z+2i|$; (b) $|z| = 2|z+1|$; (c) $|z-1| + |z+1| = 4$.

9. Find an upper bound for $|\frac{1}{z-5}|$ when z satisfies $|z-1| \leq 1$.
   Hint: Use $||z_1| - |z_2|| \leq |z_1 - z_2| \leq |z_1| + |z_2|$.

10. Show that the function $z(t) = 2e^{it}, 0 \leq t \leq 2\pi$ describes the unit circle. Sketch the curves that are given by
    (a) $z(t) = 2e^{it} + i, 0 \leq t \leq 2\pi$; (b) $z(t) = e^{(1+i)t}, 0 \leq t \leq 2\pi$