MATH305-201-2021-HW5 Homework Assignment 5 (Due Date: Feb. 22, 2022)

1. Find the region where $f(z) = Log(1 - z^3)$ is analytic.

2. Find a branch of each of the following multi-valued functions that is analytic in the given domain
   (a) $(9 + z^2)^{\frac{1}{2}}$ in $C \backslash \{x = 0, -3 \le y \le 3\}$; (b) $(z^4 - 1)^{\frac{1}{2}}$ in $\{|z| > 1\}$.

3. Find all solutions to
   (a) $\sin(z) = -i$; (b) $\sin^{-1}(i)$; $\cos(z) = 2i$; (d) $\cos^{-1}(2i)$

4. Find a solution to the boundary value problem

$$\phi_{xx} + \phi_{yy} = 0, y > 0, -1 < x < 1, y > 0$$

$$\phi(x, y) = 0, \text{ on } x = -1, y > 0; 0, \text{ on } y = 0, -1 < x < 1; 2, \text{ on } x = 1, y > 0.$$

5. Find a solution to the boundary value problem

$$\phi_{xx} + \phi_{yy} = 0, \quad x > 0, y > 0$$

$$\phi = 1 \text{ on } x = 0, y > 0; \phi_y = 0 \text{ on } 0 < x < 1, y = 0; \phi = 2 \text{ on } x > 1, y = 0$$

6. Find an inverse function for $sinh(z) = \frac{e^z - e^{-z}}{2}$ such that its value at 0 equals 0.

7. Show that $|\sin z| < 3$ when $|z| < 1$.

8. Compute the integral $\int_C f dz$ using the contour (always counter-clockwise) given
   (a) $f = x - 2xyi$; $C = \{y = x^2, 0 \le x \le 1\} \cup \{y = 1, -1 \le x \le 1\}$; (b) $f = \bar{z}^2$; C: square
   with vertices $z = 0, z = 1, z = 1 + i$ and $z = i$; (c) $f = Log(z)$; $C = \{|z| = 1, Re(z) \ge 0\}$

9. Evaluate $\int_C (z^2 + 1) dz$, where C is the following contour from $z = -i$ to $z = 1$:
   (a) the simple line segment; (b) two simple line segments, the first from $z = -i$ to $z = 0$ and
   the second from $z = 0$ to $z = 1$; (c) the circular arc $z = e^{it}, -\frac{\pi}{2} \le t \le 0$

10. Evaluate $\int_C \bar{z} dz$, where
    (a) C is the circle $|z| = 2$ traversed once counterclockwise; (b) C is the circle $|z| = 2$ traversed
    twice counterclockwise; (c) C is the circle $|z| = 2$ traversed three times clockwise.