Please replace Ex. 10 on p.18–p.19 by

Ex. 10 Solve
$$\begin{cases} x^2 u_x + xy\, u_y = u^2 \\ u = 1 \quad \text{on} \quad x = y^2 \end{cases}$$

Sol'n: Parametrize data curve:

$$x_0(\xi) = \xi^2, \quad y_0(\xi) = \xi, \quad u_0(\xi) = 1$$

$$\begin{cases} \dfrac{dx}{ds} = x^2, \quad x(0) = \xi^2 \quad \Longrightarrow \quad x = -\dfrac{1}{s+c}, \quad x(0) = \xi^2 \quad \Longrightarrow \quad x = \dfrac{\xi^2}{1-\xi^2 s} \\ \dfrac{dy}{ds} = xy, \quad y(0) = \xi \quad \Rightarrow \quad \dfrac{dy}{y} = \dfrac{\xi^2}{1-\xi^2 s} ds \quad \Rightarrow \quad y = \dfrac{\xi}{1-\xi^2 s} \\ \dfrac{du}{ds} = u^2, \quad u(0) = 1 \quad \Rightarrow \quad \dfrac{du}{u^2} = ds \Rightarrow -\dfrac{1}{u} = s + c \Rightarrow u = \dfrac{1}{1-s} \end{cases}$$

From
$$\begin{cases} x = \dfrac{\xi^2}{1-\xi^2 s} \\ y = \dfrac{\xi}{1-\xi^2 s} \end{cases} \Rightarrow \quad \xi = \frac{x}{y} \quad \Rightarrow \quad y = \frac{\frac{x}{y}}{1-\left(\frac{x}{y}\right)^2 s} \Rightarrow 1 - \frac{x^2}{y^2} s = \frac{x}{y^2}$$

$$\Rightarrow \quad y^2 - x = x^2 s \quad \Rightarrow s = \frac{y^2 - x}{x^2}$$

$$u = \frac{1}{1 - \frac{y^2 - x}{x^2}} = \frac{x^2}{x^2 + x - y^2}$$

u becomes unbounded on the curve $x^2 + x - y^2 = 0$. Reason: this is a nonlinear problem