

MATH400-101-2019 Homework Assignment 2 (Due Date: September 26, 2018, by 6pm)

Homework is admitted until 6pm on September 26, 2018. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. Find the general solutions to the following first order PDE: $yu_x + 2xyu_y = 2xu$.
2. Find the solutions to the following quasilinear problem

$$(a) \quad u_t + (1 - u)u_x = 0, t > 0, u(x, 0) = 3 - x; \quad (b) \quad u_t + u^2u_x = 0, t > 0, u(x, 0) = 1 + x$$

3. Find out the maximum breaking up $t = t_B$ for the following quasilinear problem

$$u_t + u^2u_x = 0, \quad u(x, 0) = \frac{1}{1 + x^2}$$

4. Find the expansion fan of the form $u = U(\frac{x}{t})$ for the following quasilinear problem

$$2u_t + (u^2 + 1)u_x = 0$$

5. Consider the following traffic flow problem

$$u_t + (2 - u)u_x = 0, -\infty < x < +\infty, t > 0$$

Solve the problem with

$$u(x, 0) = \frac{1}{2}, -\infty < x < +\infty$$
$$u(0-, t) = 4, \quad u(0+, t) = 1, t > 0$$

6. Consider $u(x, t)$ which satisfies

$$u_t + (u - 1)u_x = 0, -\infty < x < +\infty, t > 0$$

with

$$u(x, 0) = \begin{cases} 0, & x < 0, \\ 1, & 0 < x < 2, \\ \frac{1}{2}, & 2 < x \end{cases}$$

(a) Find the solution in different regions of the x, t plane up until the time that the expansion fan hits the characteristic curve. (b) Find the shock curve afterwards.

7. Solve the following fully nonlinear PDE:

$$u_y = u_x^2, \quad u(x, 0) = x$$

8. Solve the following fully nonlinear PDE:

$$u_x u_y = u, \quad u(x, 0) = x + 1$$