MATH400-101-2019 Homework Assignment 3 (Due Date: October 9, 2018, by 6pm)

Homework is admitted until 6pm on October 9, 2018. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (20pts) (a) State the well-posedness criteria for the following 4th order equation

$$\begin{cases} u_{tt} - u_{xxxx} = 0, t > 0, -\infty < x < +\infty, \\ u(x,0) = \phi(x), u_t(x,0) = \psi(x) \end{cases}$$

(b) Is the above problem well-posed? Try functions of the type $C(e^{mt} + e^{-mt})\sin(nx)$.

2. (20pts) For the following PDEs, determine its type and use the change of variable to transform it into one of the three standard form

(a) $u_{xx} + 3u_{xy} - 4u_{yy} = 0$, (b) $u_{xx} - 4u_{xy} + 4u_{yy} - 2u_x + u_y = 0$, (c) $u_{xx} - 2u_{xy} + 2u_{yy} = 0$

3. (10pts) Solve the following wave equation:

$$u_{tt} - u_{xx} = 0$$

$$u(x,0) = \sin x, u_t(x,0) = e^x$$

4. (20pts) For the following wave equation:

$$u_{tt} - u_{xx} = 0$$

$$u(x,0) = 0, u_t(x,0) = \begin{cases} 1, & 0 < x < 1; \\ 0, & \text{otherwise} \end{cases}$$

Find $u(x, \frac{1}{2}), u(x,1), u(x, \frac{5}{2})$.

5. (15pts) Find the solution for

$$\begin{cases} u_{tt} + 3u_{tx} - 4u_{xx} = 0, t > 0, -\infty < x < +\infty, \\ u(x,0) = \sin(x), u_t(x,0) = 0, -\infty < x < +\infty \end{cases}$$

6. (15pts) Solve the following wave equation

$$u_{tt} = c^2 u_{xx} + e^{2x}\sin(ct)$$

$$u(x,0) = \cos(x), u_t(x,0) = x + 2$$