MATH400-101 Homework Assignment 1 (Due Date: September 16 , 2019)

Homework is admitted until 6pm on September 16, 2019. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$2u_x - 3u_y = 0, u(x, 2x) = x^2 - 1$$

2. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$2yu_x + u_y = 0, u(0, y) = y$$

3. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$u_x + 2xyu_y = u, u(x, 1) = x^3, 0 \le x \le 1$$

4. (10points) Solve $u_t + 2u_x = 0$ for $x > 0, t > 0$ with $u(0, t) = e^t, t > 0$ and $u(x, 0) = x^2, x > 0$.
5. (10points) Solve $xu_t + tu_x = 0$ for $x > 0, t > 0$ with $u(0, t) = t^2, t > 0$ and $u(x, 0) = x, x > 0$.
6. (10points) Find the solution to the following first order PDE

$$2u_x - 3u_y + u = e^x, u(x, x) = x, -\infty < x < +\infty$$

7. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$xu_x + (x + y)u_y = u + x, u(x, 3x) = 0, -\infty < x < +\infty$$

8. (10points) Solve the following first order PDE and find where the solution becomes unbounded in the $x-y$ plane.

$$u_x + e^x u_y = u^2, \ u = 1 \ \text{ on the curve } y = 2e^x, 0 \le x \le 1$$

9. (10points) Find the general solutions to the following first order PDE

$$xu_x - yu_y = yu$$

10. (10points) Let $u(x, y)$ solve the first order PDE

$$yu_x + xu_y = y^3 u$$

(a). Find the general solutions. (b) Suppose we put $u = h(x)$ on $y = x$. Derive the condition that $h(x)$ must satisfy for a solution to exist.