MATH400-101-2019 Homework Assignment 4 (Due Date: October 25, 2019, by 6pm)

Homework is admitted until 6pm on October 25, 2019. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10pts) Solve $u_{tt} = c^2 u_{xx} + \cos x, u(x,0) = \cos x, u_t(x,0) = 1 + x$
2. (10pts) Solve $u_{tt} = c^2 u_{xx} + xt, u(x,0) = 0, u_t(x,0) = 0$
3. (10pts) Compute the energy for the following wave equation

$$u_{tt} = 4u_{xx}$$

$$u(x,0) = e^{-|x|},$$

$$u_t(x,0) = \begin{cases} 0, |x| > 1 \\ 1, |x| < 1 \end{cases}$$

4. (20pts) Consider the following problem

$$u_{tt} = c^2 u_{xx}, \ 0 < x < +\infty, t > 0,$$

$$u(0,t) = t^2$$

$$u(x,0) = 0, u_t(x,0) = 1.$$

(a) Use general solutions of wave equations to solve for u.
(b) Let $u = t^2 + v$ and then use the method of reflection to solve v.

5. (20pts) Consider the following wave equation:

$$u_{tt} = u_{xx}, 0 < x < 1$$

$$u(x,0) = -1, u_t(x,0) = 1, \ 0 < x < 1$$

$$u(0,t) = u(1,t) = 0$$

Use the method of reflection to find $u(\frac{1}{2}, \frac{5}{4})$.

6. (10pts) Find the ordinary differential equation satisfied by f, if the function

$$u(x,t) = t^2 f(\frac{x}{\sqrt{t}})$$

satisfies $u_t = u_{xx}$.

7.(10pts) Solve

$$u_t = ku_{xx}, -\infty < x < \infty$$

$$u(x,0) = e^{-|x|}, -\infty < x < +\infty$$

Write the solution in terms of $\mathcal{E}rf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-p^2} dp$.

8.(10pts) Solve

$$u_t = ku_{xx}, -\infty < x < \infty$$

$$u(x,0) = \begin{cases} 0 \ x < -1, \\ 2, -1 < x < 1, \\ 1, x > 1 \end{cases}$$

Write the solution in terms of $\mathcal{E}rf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-p^2} dp$.