**MATH400-101-2022 Homework Assignment 2 (Due Date: September 26, 2022, by 1am)**

Homework is admitted until 1am on September 26, 2022. You can submit either at my office by 6pm on Friday or at Canvas. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10points) Solve the following first order PDE and find where the solution becomes unbounded in the $x-y$ plane.

$$u_x + e^x u_y = u^2, \ u = 1 \ \text{ on the curve } y = 2e^x, 0 \leq x \leq 1$$

2. (5pts) Find the general solutions to the following first order PDE: $u_x + yu_y = 3u + y$.

3. (10points) Find the general solutions to the following first order PDE

$$xu_x - yu_y = yu$$

4. (10points) Let $u(x, y)$ solve the first order PDE

$$yu_x + xu_y = y^3 u$$

(a). Find the general solutions. (b) Suppose we put $u = h(x)$ on $y = x$. Derive the condition that $h(x)$ must satisfy for a solution to exist.

5. (20pts) Find the solutions to the following quasilinear problem

$$(a)(5) \ \ u_t + uu_x = 0, t > 0, u(x, 0) = x - 3; \quad (b)(5) \ \ u_t + (1 - u)u_x = 0, t > 0, u(x, 0) = 1 - 3x, 0 \leq x \leq 1$$

$$(c)(10) \ \ u_t + u^2 u_x = 0, t > 0, u(x, 0) = x$$

6. (15pts) Find the solutions to the following quasilinear problem

$$(a)(5) \ \ u_t + uu_x = 0, u(0, t) = 1, 1 < t < 2; \quad (b)(10) \ \ u_t + uu_x = 0, t > 0, u(0, t) = t, t > 0$$

7. (10pts) Find out the maximum breaking up $t = t_B$ for the following quasilinear problem

$$(a) \ \ u_t + uu_x = 0, \ u(x, 0) = \frac{1}{1 + x^2} : \quad (b) u_t + u^2 u_x = 0, u(x, 0) = e^{-x^2}$$

8.(10pts) Find the expansion fan of the form $u = U(\frac{x}{t})$ for the following quasilinear problem

$$(a) \ \ u_t + 2u^2 u_x = 0; \quad (b) u_t + (2u - u^2)u_x = 0$$

9.(10pts) Use expansion fan to solve the following quasilinear first order problem

$$u_t + u^3 u_x = 0;$$

$$u(x, 0) = \begin{cases} -1, -\infty < x < 1; \\ 1, x > 1 \end{cases}$$