MATH400-101-2022 Homework Assignment 3 (Due Date: October 2, 2022, by 11pm)

Homework is admitted until 11pm on October 2, 2022. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10pts) Solve the following quasilinear first order problem by inserting a shock curve:

$$u_t + (u-1)u_x = 0, t > 0$$

$$u(x,0) = \begin{cases} 2, -\infty < x < 1; \\ 1, 1 < x + \infty \end{cases}$$

2. (10pts) Solve the following quasilinear first order problem:

$$u_t + uu_x = 0;$$

$$u(x,0) = \begin{cases} 0, -\infty < x < 0; \\ 1, 0 < x < 1; \\ -1, x > 1 \end{cases}$$

Find the solution in different regions of the x, t plane up until the time that **the shock curve hits the expansion fan**.

3. (20pts) Consider $u(x,t)$ which satisfies

$$u_t + (u-1)u_x = 0, \ -\infty < x < +\infty, t > 0$$

with

$$u(x,0) = \begin{cases} 0, \ x < 0, \\ 2, \ 0 < x < 2, \\ 1, \ 2 < x \end{cases}$$

(a) Find the solution in different regions of the x, t plane up until the time that **the shock curve hits the expansion fan**. (b) Find the shock curve afterwards.

4. (20pts) Consider the following traffic flow problem

$$u_t + (2-u)u_x = 0, -\infty < x < +\infty, \ t > 0$$

Solve the problem with

$$u(x,0) = \frac{1}{2}, -\infty < x < +\infty$$

$$u(0-,t) = 5, \ u(0+,t) = 1, t > 0$$

5. (20pts) Consider the following first order PDE:

$$u_t + (2-u)u_x = 0$$

$$u(x,0)=\begin{cases} 2, & x<0 \\ 1, & x>0 \end{cases}$$

$$u(0-,t)=3, t>0$$

(a). Find the expansion fan solution. (b). Find the solution.

6. (10pts) Solve the following fully nonlinear PDE:

$$u_y=u_x^2, \quad u(x,0)=x$$

7. (10pts) Consider the following fully nonlinear PDE:

$$u_x u_y=u, \ u(x,0)=x+1$$

(a) Compute the Charpit's equation. (b) Find the solution.