MATH400-101-2022 Homework Assignment 5 (Due Date: October 25th, 2022, by 11pm)

Homework is admitted until 11pm on **October 25th**, 2022. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10pts) Compute the energy for solution to the following problem

$$u_{tt} = 4u_{xx}, -\infty < x < +\infty,$$
$$u(x,0) = u_t(x,0) = \begin{cases} 1-|x|, & |x| \leq 1; \\ 0, & |x| > 1 \end{cases}$$

2. (20pts) Solve the following wave equation

$$u_t = u_{xx} + \frac{2}{x}u_x, x > 0, t > 0$$
$$u(x,0) = \phi(x), u_t(x,0) = \psi(x)$$

by the following method:

(a) Let $U = xu$. Show that $U(x,t)$ satisfies

$$U_t = U_{xx}, x > 0, t > 0$$
$$U(x,0) = x\phi(x), U_t(x,0) = x\psi(x)$$
$$U(0,t) = 0, t > 0$$

(b) Solve U and then find $u(x,t)$.

(Note: this is the wave equation in R^3 in radial coordinate)

3. (10pts) Solve $u_{tt} = 9u_{xx} + \sin x, u(x,0) = 0, u_t(x,0) = e^x$
4. (10pts) Solve $u_{tt} = 4u_{xx} + xt, u(x,0) = 0, u_t(x,0) = 0$
5. (20pts) Consider the following problem

$$u_{tt} = c^2 u_{xx} + 1, \ 0 < x < +\infty, t > 0,$$
$$u(0,t) = 0, t > 0$$
$$u(x,0) = 0, u_t(x,0) = x.$$

Use the method of reflection to solve u.

6. (10pts) Consider the following wave equation:

$$u_{tt} = u_{xx}, 0 < x < 1$$
$$u(x,0) = -1, u_t(x,0) = 1, \ 0 < x < 1$$
$$u(0,t) = u(1,t) = 0$$

Use the method of reflection to find $u(\frac{1}{2}, \frac{5}{4})$.

7. (10pts) Find the ordinary differential equation satisfied by f, if the function

$$u(x,t) = t^3 f(\frac{x}{\sqrt{t}})$$

satisfies $u_t = u_{xx}$.

8.(10pts) Solve

$$u_t = ku_{xx}, -\infty < x < \infty$$
$$u(x,0) = \begin{cases} -1, -1 < x < 0; \\ 2, 1 < x < 2; \\ 0, \ \text{otherwise} \end{cases}$$

Write the solution in terms of $Q(x,t) = \mathcal{E}rf(\frac{x}{\sqrt{t}}) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{t}}} e^{-p^2} dp.$